Towards Feasible Implementations of Low-Latency Multi-Writer Atomic Registers

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Abstract—This work explores implementations of multi-writer/multi-reader (MWMR) atomic registers in asynchronous, crash-prone, message-passing systems with the focus on low latency and computational feasibility. The efficiency of atomic read/write register implementations is traditionally measured in terms of the latency of read and write operations. To reduce operation latency researchers focused on the communication costs, expressed as the number of communication round-trips (or rounds), often ignoring the computation costs.

In this paper we consider efficiency of a register implementation in terms of both communication and computation costs. As of this writing, algorithm $SFW$ is the sole known MWMR algorithm that allows single round read and write operations. The algorithm uses sets of intersecting quorums, and to enable single round operations, $SFW$ relies on the evaluation of certain predicates. We formulate a new combinatorial problem that captures the computational burden of evaluating the predicates in algorithm $SFW$ and we show that it is NP-Complete. To make the evaluation of the predicates feasible, we present a polynomial log-approximation algorithm for this problem and we show how to use it with algorithm $SFW$. Then we present a new algorithm, called $CWF_k$, that allows fast operations independently of the quorum system construction. The algorithm implements two-round writes and allows reads to complete in a single round. We conclude with experimental evaluations of our algorithms obtained from simulations in NS2.

Keywords—Atomic Memory; NP-Complete; Approximation Algorithms; MWMR Registers;

I. INTRODUCTION

Emulating atomic registers in asynchronous, crash-prone, message-passing systems is one of the basic problems in distributed computing. In such settings the register is replicated among a set of replica hosts or servers to provide fault-tolerance and availability. Then read and write operations are implemented as communication protocols that ensure atomic consistency.

Efficiency of register implementations is normally measured in terms of the latency of read and write operations. Two factors affect operation latency: (a) computation, and (b) communication delays. An operation may need to communicate with servers to read or write the register value. This involves at least a single communication round-trip, or round, i.e., messages from the invoking process to some servers and then the replies from these servers. Previous

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client side decision tools, called Quorum Views, developed for the SWMR setting in [9], we derive algorithm CWFR. The new algorithm uses the conventional two round writes. To allow fast read operations the algorithm analyzes, using quorum views, the distribution of a value within a quorum of replies from servers. As multiple writes can occur concurrently, an iterative technique is used to discover the latest potentially completed write operation.

(3) We obtained experimental results by simulating our algorithms on the NS2 simulator. In particular, we first compare algorithms SFW and APRX-SFW in terms of the number of second communication rounds and show that the experimental results are within the theoretical approximation bounds. Furthermore, the hardness of the predicate evaluation computation is made evident from the observed operation latency (as the number of servers increases). We then compare the operation latency of algorithms APRX-SFW, CWFR, and a traditional two-round algorithm that incurs a low computational overhead. We observe that the first two algorithms achieve lower latency despite the computational burden. Finally, we compare the operation latency and the percentage of fast reads of algorithms CWFR and APRX-SFW. We observe that in quorum systems with small intersection degree, CWFR seems to perform better than APRX-SFW; in quorums with large intersection degree APRX-SFW performs better.

Background and prior work: Attiya et al. [2] gave a SWMR algorithm that achieves consistency by using intersecting sets of servers in combination with \( (\text{timestamp}, \text{value}) \) value tags. A write operation increments the writer’s local timestamp and delivers the new tag-value pair to a majority of servers, taking one round. A read operation obtains tag-value pairs from some majority, then propagates the pair corresponding to the highest timestamp to some majority of servers, thus taking two rounds.

The majority-based approach in [2] is readily generalized to quorum-based approaches in the MWMR setting (e.g., [14], [13], [8], [11]). Such algorithms require at least two communication rounds for each read and write operation. Both write and read operations query the servers for the latest value of the replica during the first round. In the second round the write operation generates a new tag and propagates the tag along with the new value to a quorum of servers. A read operation propagates to a quorum of servers the largest value it discovers during its first round. Dolev et al. [5] and Chockler et al. [3], provide MWMR implementations where some reads involve a single communication round when it is confirmed that the value read was already propagated to some quorum.

Dutta et al. [6] present the first fast atomic SWMR implementation where all operations take a single communication round. They show that fast behavior is achievable only when the number of reader processes \( R \) is inferior to \( \frac{S}{t} - 2 \), where \( S \) the number of servers, \( t \) of whom may crash. They also showed that fast MWMR implementations are impossible even in the presence of a single server failure. Georgiou et al. [10] introduced the notion of virtual nodes that enables an unbounded number of readers. They define the notion of semifast implementations where only a single read operation per write needs to be “slow” (take two rounds). They also show the impossibility of semifast MWMR implementations.

Georgiou et al. [9] showed that fast and semifast quorum-based SWMR implementations are possible iff a common intersection exists among all quorums. Hence a single point of failure exists in such solutions (i.e., any server in the common intersection), making such implementations not fault-tolerant. To trade efficiency for improved fault-tolerance, weak-semifast implementations in [9] allow more than a single slow read per write operation, and all writes to be fast. To obtain a weak-semifast implementation they introduced a client-side decision tool called Quorum Views that enables fast read operations under read/write concurrency when general quorum systems are used.

Recently, Englert et al. [7] developed an atomic MWMR register implementation, called algorithm SFW, that allows both reads and writes to complete in a single round. To handle server failures, their algorithm uses \( n \)-wise quorum systems: a set of subsets of servers, such that each \( n \) of these subsets intersect. The parameter \( n \) is called the intersection degree of the quorum system. The algorithm relies on \( \langle \text{tag}, \text{value} \rangle \) pairs to totally order write operations. In contrast with traditional approaches, the algorithm uses the server side ordering (SSO) approach that transfers the responsibility of incrementing the tag from the writers to the servers. This way, the query round of write operations is eliminated. The authors proved that fast MWMR implementations are possible if and only if they allow not more than \( n-1 \) successive write operations, where \( n \) is the intersection degree of the quorum system. If read operations are also allowed to modify the value of the register then from the provided bound it follows that a fast implementation can accommodate up to \( n - 1 \) readers and writers.

Paper organization: In Section II we give the model of computation and the notation we use throughout. In Section III we overview algorithm SFW. Section IV introduces the new combinatorial problem, its analysis, and the approximation algorithm. Algorithm CWFR is presented in Section V. Simulation results and comparisons of algorithms are in Section VI. We conclude in Section VII. Omitted discussion and proofs are found in [1].

II. Model and Definitions

We consider the asynchronous message-passing model. There are three distinct finite sets of crash-prone processors: a set of readers \( R \), a set of writers \( W \), and a set of servers \( S \). The identifiers of all processors are unique and comparable. Communication among the processors is accomplished via reliable communication channels.

Servers and quorums: Servers are arranged into intersecting sets, or quorums, that together form a quorum system \( Q \). For a set of quorums \( A \subseteq Q \) we denote the
intersection of the quorums in \( A \) by \( I_A = \bigcap_{Q \in A} Q \). A quorum system \( Q \) is called an \( n \)-wise quorum system if for any \( A \subseteq Q \), s.t. \( |A| = n \) we have \( I_A \neq \emptyset \). We call \( n \) the intersection degree of \( Q \). Any quorum system is a 2-wise (pairwise) quorum system because any two quorums intersect. At the other extreme, a \( |Q| \)-wise quorum system has a common intersection among all quorums. Note that an \( n \)-wise quorum system is also a \( k \)-wise quorum system, for \( 2 \leq k \leq n \).

Our system allows processes to fail by crashing. A process \( i \) is faulty in an execution if \( i \) crashes in the execution (once a process crashes, it does not recover); otherwise \( i \) is correct. A quorum \( Q \in Q \) is non-faulty if for \( \forall i \in Q \), \( i \) is correct; otherwise \( Q \) is faulty. We assume that at least one quorum in \( Q \) is non-faulty in any execution.

**Atomicity:** We study atomic read/write register implementations, where the register is replicated at servers. Reader \( p \) requests a read operation \( \rho \) on the register using action \( \text{read}_{\rho} \). Similarly, a write operation is requested using action \( \text{write}^{(*)}\rho \) at writer \( p \). The steps corresponding to such actions are called invocation steps. An operation terminates with the corresponding acknowledgment action; these steps are called response steps. An operation \( \pi \) is incomplete in an execution when the invocation step of \( \pi \) does not have the associated response step; otherwise we say that \( \pi \) is complete. Requests made by read and write processes are well-formed: a process does not request a new operation until it receives the response for a previously invoked operation.

In an execution, we say that an operation (read or write) \( \pi_1 \) precedes another operation \( \pi_2 \), or \( \pi_2 \) succeeds \( \pi_1 \), if the response step for \( \pi_1 \) precedes in real time the invocation step of \( \pi_2 \); this is denoted by \( \pi_1 \rightarrow \pi_2 \). Two operations are concurrent if neither precedes the other.

Correctness of an implementation of an atomic read/write object is defined in terms of the atomicity and termination properties. Assuming the failure model discussed earlier, the termination property requires that any operation invoked by a correct process eventually completes. Atomicity is defined as follows [12]. For any execution if all read and write operations that are invoked complete, then the operations can be partially ordered by an ordering \( \prec \), so that the following properties are satisfied:

**P1.** The partial order is consistent with the external order of invocation and responses, that is, there do not exist operations \( \pi_1 \) and \( \pi_2 \), such that \( \pi_1 \rightarrow \pi_2 \), yet \( \pi_2 \prec \pi_1 \).

**P2.** All write operations are totally ordered and every read operation is ordered with respect to all the writes.

**P3.** Every read operation ordered after any writes returns the value of the last write preceding it in the partial order, and any read operation ordered before all writes returns the initial value of the register.

**Efficiency and Fastness:** We measure the efficiency of an atomic register implementation in terms of computation and communication round-trips (or simply rounds). A round is defined as follows [6], [10], [9]:

**Definition 2.1:** Process \( p \) performs a communication round during operation \( \pi \) if all of the following hold:

1. \( p \) sends request messages for \( \pi \) to a set of processes,
2. any process \( q \) that receives a request message from \( p \) for operation \( \pi \), replies without delay.
3. when process \( p \) receives enough replies it terminates the round (either completing \( \pi \) or starting a new round).

Operation \( \pi \) is fast [6] if it completes after its first communication round; an implementation is fast if in each execution all operations are fast. We use quorum systems and tags to maintain and impose an ordering on the values written to the register replicas. We say that a quorum \( Q \in Q \), replies to a process \( p \) for an operation \( \pi \) during a round, if \( \forall s \in Q \), \( s \) receives a message during the round and replies to this message, and \( p \) receives all such replies.

Given that any subset of readers or writers may crash, the termination of an operation cannot depend on the progress of any other operation. Furthermore we guarantee termination only if servers’ replies within a round of some operation do not depend on receipt of any messages sent by other processes. Thus we can construct executions where only the messages from the invoking processes to the servers, and from the servers to the invoking processes are delivered. Lastly, to guarantee termination under the assumed failure model, no operation can wait for more than a single quorum to reply within any a single round.

**III. BRIEF DESCRIPTION OF ALGORITHM SFW**

Algorithm SFW assumes that the servers are arranged in an \( n \)-wise quorum system. To order the written values the algorithm uses \( \langle \text{tag}, \text{value} \rangle \) pairs. To enable fast writes the algorithm assigns partial responsibility to the servers for the ordering of the values written. If a server receives a write request it generates a new tag, larger than any of the tags it witnessed, and assigns it to the value enclosed in the write message. The server records a generated tag, along with the write operation it was created for, in a set called \( \text{inprogress} \). The set holds only the latest tag generated for each writer.

Each reader or writer must communicate with a quorum of servers, say \( Q \), during the first round of each read/write operation. Due to concurrency different servers can receive messages from write operations in different order, thus an operation may witness different tags assigned to a single write operation. To deal with this algorithm SFW uses two predicates to determine whether “enough” servers in the replying quorum assigned the same tag to a particular write operation. Let \( n \) be the intersection degree of the quorum system, and \( \text{inprogress}_s(\pi) \) be the \( \text{inprogress} \) set that server \( s \) enclosed in the message it sent to the process that invoked operation \( \pi \). The write and read predicates are:

**PW:** Writer predicate for a write \( \omega \): \( \exists \tau, A, MS \) where: \( \tau \in \{\langle \omega \rangle : \langle \omega \rangle \in \text{inprogress}_s(\omega) \wedge s \in Q \}, A \subseteq Q, 0 \leq |A| \leq \frac{n}{2} - 1, \) and \( MS = \{s : s \in Q \wedge \tau \in \text{inprogress}_s(\omega)\}, \) s.t. either \( |A| \neq 0 \) and \( I_A \cap Q \subseteq MS \) or \( |A| = 0 \) and \( Q = MS \).
PR: Reader predicate for a read $\rho$: $\exists \tau, B, MS$, where:

$$\max(\tau) \in \bigcup_{i \in Q} \text{inprogress}_i(\rho), B \subseteq Q, 0 \leq |B| \leq \frac{n}{2} - 2,$$

and $MS = \{s : s \in Q \land \tau \in \text{inprogress}_i(\rho)\}$, s.t. either $|B| \neq 0$ and $I_B \cap Q \subseteq MS$ or $|B| = 0$ and $Q = MS$.

The predicates examine whether the same tag for a write operation is contained in the replies of all servers in the intersection among the replying quorum and $\frac{n}{2} - 1$ for PW (resp. $\frac{n}{2} - 2$ for PR) of other replying quorums. Satisfaction of the predicates for a tag $\tau$ guarantees that any subsequent operation will also determine that the write operation is assigned tag $\tau$. If the predicates hold with $|A| \geq \frac{n}{2} - 1$ or $|B| = \frac{n}{2} - 2$ then the write or read operation respectively needs to proceed to a second round. A write operation can only be fast if PW holds. A read operation can be fast even if PR does not hold, but the read observed enough confirmed tags with the same value. Confirmed tags are maintained in the servers and they indicate that either the write of the value with that tag is complete, or the tag was returned by some read operation. See [7] for full details.

IV. NP-COMPLETENESS AND APPROXIMATION

The complexity of the predicates raises the question whether they can be computed efficiently. The two predicates can be captured by a decision problem that we formalize as follows:

**Definition 4.1 (k-SET-INTERSECTION):** Given a set of elements $U$, a subset of those elements $M \subseteq U$ and a set of subsets $Q = \{Q_1, \ldots, Q_n\}$ s.t. $Q_i \subseteq U$, a set $I$ is an intersecting set if $I \subseteq Q_i, \bigcap_{Q \in I} Q \neq \emptyset$, and $\bigcap_{Q \in I} Q \subseteq M$. If $|I| = k$ then $I$ is a $k$ intersecting set.

To the best of our knowledge this is a new combinatorial problem and it is similar to the open problem stated in [4]. In the context of [7], the universe of elements $U$ is the set of servers, and the set of subsets of $U$ is the deployed quorum system. Clearly $k$-SET-INTERSECTION is in NP: given $(U, M, Q)$ and a set $I \subseteq Q$, s.t. $|I| = k$, we can verify in polynomial time (with respect to $|Q|$) if $\bigcap_{Q \in I} Q \subseteq M$.

A. Polynomial Reduction from 3-SAT

We now show that the $k$-SET-INTERSECTION problem is NP-Complete by providing a polynomial reduction from the 3-SAT problem. The reduction involves a polynomial transformation of the input to 3-SAT to an instance of k-SET-INTERSECTION. We first provide the definition of 3-SAT [15]:

**Definition 4.2 (3-SAT):** Let $X = \{x_1, \ldots, x_n\}$ be a set of variables and $\Phi$ a boolean formula in CNF (Conjunctive Normal Form) where each clause contains at most three literals (variable or its negation). Is there a truth assignment to every $x_i \in X$ s.t. $\Phi$ becomes true?

**Construction:** We transform an instance of the 3-SAT problem to an instance $(U, M, Q, k)$ of k-SET-INTERSECTION as follows. Let $k = n$ the total number of variables. The universe consists of an element for each variable, the negation of each variable, and an element for each clause $C_i$ of 3-SAT. It also includes $n$ elements which will ensure that each variable is chosen at least once:

$$U = \{x_1, \ldots, x_n, \overline{x}_1, \ldots, \overline{x}_n, C_1, \ldots, C_m, \ell_1, \ldots, \ell_r\}$$

The set $M \subseteq U$ contains all the elements that appear in the clauses. Both the variable $x_i$ and its negation $\overline{x}_i$ may appear in $M$, if they appear in any clause of the boolean formula. Thus the set $M$ is constructed in $O(2nm)$ time as follows:

$$M = \{x_i : \exists C_j, \overline{x}_i \in C_j\} \cup \{x_i : \exists C_j, \overline{x}_i \not\in C_j\}$$

Lastly we construct the set of subsets $Q$. For each variable $x_i \in M$ we construct a subset, $Q_i$, and for each variable $\overline{x}_i \in M$ we construct a subset $Q'_i$. Every $Q_i$ contains the variable $x_i$, the variables $x_j, j \neq i$ and their negations, and the clauses that do not contain $x_i$ or contain $\overline{x}_i$. Intuitively, those are the clauses that are not directly satisfied if we set $x_i = \text{true}$. Finally, we include one element $\ell_j$ for each $j \neq i$. These sets will ensure that for a variable $x_i$ we choose either $Q_i$ or $Q'_i$, but not both. We construct $Q'_i$ similarly for $\overline{x}_i$. More formally the sets we obtain are the following:

$Q_i = \{x_i \in M\} \cup \{x_j, \overline{x}_i : j \neq i\} \cup \{C_j : x_i \not\in C_j\} \cup \{\ell_j : j \neq i\}$

$Q'_i = \{\overline{x}_i \in M\} \cup \{x_j, \overline{x}_i : j \neq i\} \cup \{C_j : \overline{x}_i \not\in C_j\} \cup \{\ell_j : j \neq i\}$

Given the above sets, the set of subsets $Q$ is: $Q = \{\bigcup_{x_i \in M}(Q_i)\} \cup \{\bigcup_{\overline{x}_i \in M}(Q'_i)\}$. The construction of all sets $Q_i$ and $Q'_i$ takes at most $O(2n^2m)$.

The idea of this construction is to find a set of subsets such that their intersection contains positive and negative variables and no clauses or elements $\ell_j$. In our construction this implies that setting the variables of the intersection to true satisfies all clauses. In addition, the elimination of the elements $\ell_j$, in combination with $k$ being equal to $n$, implies that we choose either $Q_i$ or $Q'_i$ but not both. Therefore, the intersection of $n$ subsets implies that we chose a single truth value for every variable. With this construction we formally show that 3-SAT $\leq_p$ k-SET-INTERSECTION, obtaining the following theorem:

**Theorem 4.3: k-SET-INTERSECTION is NP-Complete.**

B. Approximation Algorithm

Here we provide a polynomial time algorithm that yields an approximate solution to the problem given in Definition 4.1. As a part of our algorithm we use the standard SET-COVER greedy log-approximation algorithm (cf. [15]). The set cover problem is defined as follows [15]:

**Definition 4.4 (SET-COVER):** Given a universe $U$ of elements, a collection of subsets of $U$, $S = \{S_1, \ldots, S_z\}$, and a number $k$, find at most $k$ sets of $S$ such that their union covers all elements in $U$.

We now present the steps of the algorithm in Figure 1 and provide an explanation of the algorithm’s rationale.

Every $T_m$ contains the complements of the quorums that contain $m$. Let $R_{m,i} = (U - M) \cap (Q_i - M)$ for $m \in$
For an instance \((U, M, Q, k)\) of \(k\)-SET-INTERSECTION do:

**Step 1:** \(\forall m \in M\), let \(T_m = \{(U - M) - (Q_i - M) : m \in Q_i\}\)

**Step 2:** Run SET-COVER greedy algorithm on the instance \(\{U - M, T_m, k\}\) for every \(m \in M\):

- **Step 2a:** Pick the set \(R_i \in T_m\) with the maximum uncovered elements
- **Step 2b:** Take the union of every \(R_i \in T_m\) picked in Step 2a (incl. \(R_i\))
- **Step 2c:** If the union equals \(U - M\) go to Step 3; else if there are more sets in \(T_m\) go to Step 2a else repeat for another \(m \in M\)

**Step 3:** For any set \((U - M) - (Q_i - M)\) in the solution of set cover, add \(Q_i\) in the intersecting set.

Figure 1. Polynomial approximation algorithm for \(k\)-SET-INTERSECTION.

Given the sets \(R_{m,i}\) if we can find \(k\) of those that \(R_{m,1} \cup \cdots \cup R_{m,k} = U - M\), then by de Morgan’s Law it follows that \(R_{m,1} \cap \cdots \cap R_{m,k} = \emptyset\). Since, \(R_{m,i} = (U - M) - (Q_i - M)\), then \(R_{m,i} = (Q_i - M)\) and \(R_{m,1} \cap \cdots \cap R_{m,k} = (Q_1 - M) \cap \cdots \cap (Q_k - M) = \emptyset\) \((1)\)

By construction \(\forall R_{m,i} \in T_m\), \(m \in Q_i\), and thus \(\{m\} \subseteq Q_i \cap \cdots \cap Q_k\). From this and \((1)\) it follows that \(Q_i \cap \cdots \cap Q_k\) is a non-trivial subset of \(M\).

It is known [15] that SET-COVER greedy algorithm is a log \(u\)-approximation algorithm, where \(u = \|U\|\). That is, if \(k\) is the optimal solution, then the greedy algorithm will include at most \(k \log u\) sets in its solution. As the number of subsets in the solution of \(k\)-SET-INTERSECTION is the same as the number of subsets in the solution of SET-COVER, we obtain the following lemma:

**Lemma 4.5:** The algorithm in Figure 1 is a log \(u\)-approximation algorithm for the \(k\)-SET-INTERSECTION problem, where \(u = \|U\|\).

If we use the above algorithm to evaluate the predicates of algorithm SFW, the resulting implementation yields a logarithmic in the number of servers increase in the number of second communication rounds. This is a modest price to pay in exchange for substantial reduction in the computation overhead of algorithm SFW. In Section VI we present an empirical evaluation of the approximate algorithm SFW comparing it to the original algorithm SFW.

V. ALGORITHM CWFR

In this section we explore the possibility of introducing fast operations in the MWMR setting when servers are organized as an arbitrary quorum system. We introduce a new algorithm, called algorithm CWFR, that enables fast read operations by adopting the general idea of Quorum Views [9]. The algorithm employs two techniques:

- (i) the typical query and propagate approach (two rounds) for write operations, and
- (ii) analysis of Quorum Views [9] for potentially fast (single round) read operations.

Read operations can be fast in algorithm CWFR even when the value returned is not yet propagated to a full quorum. This distinguishes algorithm CWFR from previous approaches [5], [3]. To impose a total ordering on the written values, algorithm CWFR uses \((tag, value)\) pairs. A tag is a tuple of the form \((\tau, w)\) \(\in \mathbb{N} \times \mathcal{W}\), where \(\tau\) is the timestamp and \(w\) is a writer identifier. Such tags are compared lexicographically.

A. Quorum Views

We generalize the definition of quorum views from [9] for use with structured tags:

**Definition 5.1:** Let process \(p\) receive replies from every server \(s\) in some quorum \(Q \in \mathcal{Q}\) for a read or write operation \(\pi\). Let a reply from \(s\) include a tag \(\text{tag}_s(\pi)\) and let \(\text{maxTag} = \max_{s \in Q}(\text{tag}_s(\pi))\). We say that \(p\) observes one of the following quorum views for \(Q\):

1. \(qV\text{iew} (1)\): \(\forall s \in Q : \text{tag}_s(\pi) = \text{maxTag}\),
2. \(qV\text{iew} (2)\): \(\forall Q' \in Q : Q \neq Q' \land \exists A \subseteq Q \cap Q', s.t. A \neq \emptyset\) and \(\forall s \in A : \text{tag}_s(\pi) < \text{maxTag}\),
3. \(qV\text{iew} (3)\): \(\exists s' \in Q : \text{tag}_{s'}(\pi) < \text{maxTag}\) and \(\exists Q' \in Q s.t. Q \neq Q' \wedge \forall s \in Q \cap Q' : \text{tag}_s(\pi) = \text{maxTag}\)

Restating the above definition, \(qV\text{iew} (1)\) requires that all servers in some quorum reply with the same tag. \(qV\text{iew} (2)\) reveals that some servers in the quorum contain an older value, but there exists an intersection where all of its servers contain the new value. Finally \(qV\text{iew} (3)\) is the negation of the other two views, revealing a quorum where the new value is neither distributed to the full quorum nor distributed fully in any of its intersections.

B. Description of CWFR

The original quorum views algorithm [9] relies on the fact that there is a single writer. If a quorum view is able to predict the non-completeness of the latest write operation, it is immediately understood that – by the well-formedness of the single writer – any previous write operation is already complete. Multiple writers invalidate such a conclusion: different values (and tags) may be written concurrently. Hence, the discovery of a write operation that propagates some tag does not imply the completion of the write operations that propagate a smaller tag. Thus a direct adaptation of the quorum view idea from the SWMR model to the MWMR model is not possible. Consequently, algorithm CWFR incorporates an iterative technique around quorum views that not only predicts the completion status of a write operation, but also detects the last potentially complete write operation. Below we provide a description of our algorithm and present the main idea behind our technique. The pseudocode of the algorithm appears in Figure 2.

**Writers:** The write protocol has two rounds. During the first round the writer discovers the maximum tag among the servers; it sends read messages to all servers and waits for replies from all members of some quorum. It then discovers the maximum tag among the replies and generates a new tag in which it encloses the incremented timestamp of the maximum tag, and the writer’s identifier. In the second round, the writer associates the value to be written with the new tag, it propagates the pair to some quorum, and completes the write.
write(val):
  init: tag=0, wid=1, v=⊥, wcounter=0
  1: wcounter++
  2: send (READ, (tag, v), wcounter) to all servers
  3: wait for the servers of a quorum Q to reply
  4: /* find maximum tag among the replies */
  5: tag = max_{i \in Q} (s.tag)
  6: /* increment the maximum tag and generate a new tag */
  7: tag = (tag+1, wid)
  8: v = val
  9: wcounter++
  10: send (WRITE, (tag, v), wcounter) to all servers
  11: wait for the servers of a quorum Q to reply
  12: return OK

read():
  init: tag=0, wid=1, rcounter=0
  1: rcounter++
  2: send (READ, (tag, v), wcounter) to all servers
  3: wait for the servers of a quorum Q to reply
  4: while (Q ≠ ∅) do
  5:  (maxTag, v) = max_{i \in Q} (s.tag, s.v)
  6:  if (∃ s \in Q : s.tag = maxTag) then
  7:      /* qView(1) */
  8:      tag = maxTag
  9:      return tag
  10:  end if
  11:  /* qView(2) */
  12:  if (∀ s \in Q \& s.tag < maxTag) then
  13:      tag = maxTag
  14:      send (WRITE, (tag, v), wcounter) to all servers
  15:      wait for the servers of a quorum Q to reply
  16:      return tag
  17:  end if
  18:  /* qView(3) */
  19:  if (∀ s \in Q : s.tag = maxTag) then
  20:      tag = maxTag
  21:      end if
  22: end while

serve():
  init: tag=0, wid=1, pCounter=0
  1: upon receipt of (msgType, (t, val), counter) from process p
  2: /* check message freshness */
  3: if counter > pCounter then
  4:      (tag, v) = (t, val)
  5:      end if
  6: if msgType = WRITE then
  7:      send (WRITEACK, (tag, v), pCounter[p]) to p
  8: else
  9:      send (READACK, (tag, v), pCounter[p]) to p
  10: end if
  11: end if
  12: end if

Figure 2: Pseudocode for Writer, Reader and Server of algorithm CWFk.

Readers: The read protocol is more involved. The reader sends a read message to all servers and waits for some quorum to reply. Once a quorum replies, the reader determines maxTag. Then the reader analyzes the distribution of the tag within the responding quorum Q in an attempt to determine the latest, potentially complete, write operation. This is accomplished by determining the quorum view conditions. Conditions of qView(1) and qView(3) are straightforward. When condition for qView(1) is detected, the read completes and the value associated with the discovered maxTag is returned. In the case of qView(3) the reader continues to the second round, advertising the latest tag (maxTag) and its associated value. When a full quorum replies in the second round, the read returns the value associated with maxTag.

Analysis of qView(2) involves the discovery of the earliest completed write operation. This is done iteratively by (locally) removing the servers from Q that replied with the largest tags. After each iteration the reader determines the next largest tag in the remaining server set, and then re-examines the quorum views in the next iteration. This process eventually leads to either qView(1) or qView(3) being observed. If qView(1) is observed, then the read completes in a single round by returning the value associated with the maximum tag among the servers that remain in Q. If qView(3) is observed, then the reader proceeds to the second round as above, and upon completion it returns the value associated with the maximum tag maxTag discovered among the original respondents in Q.

Servers: The servers play a passive role. They receive read or write requests, update their object replica accordingly, and reply to the process that invoked the operation. Upon receipt of any message, the server compares its local tag with the tag included in the message. If the tag of the message is higher than its local tag, the server adopts the higher tag along with its corresponding value. Once this is done the server replies to the invoking process.

Main Idea: We now explain the idea behind our technique. Observe that under our failure model, any write operation can expect a response from at least one full quorum. Moreover a write ω distributes its tag tagω to some quorum, say Qω, before completing. Thus, when a read operation ρ, s.t. ω → ρ, receives replies from some quorum Qj, it observes one of the following tag distributions: (a) if Qj = Qω, then ∀s ∈ Qj, tagω = tagω (qView(1)), or (b) if Qj ≠ Qω, then ∀s ∈ Qj, tagω = tagω (qView(3)). Hence, if ρ observes a distribution as in qView(1) then the write operation potentially completed and received replies from the same quorum that replied to ρ. Alternatively, if only an intersection contains a uniform tag (i.e., the case of qView(3)) then there is a possibility that the write completed in an intersecting quorum (in this example Qj). The read operation is fast in qView(1) since it is determinable that the write potentially completed. The read proceeds to the second round in qView(3), since the completion of the write is indeterminable and it is necessary to ensure that any subsequent operation observes that tag. If neither qView(1) nor qView(3) hold, then qView(2) holds, and it must be the case that the write that yields the maximum tag is not yet complete. Hence we try to discover the latest potentially complete write by removing all servers with the highest tag from Qj and repeating the analysis. If at some iteration, qView(1) holds on the remaining tag values, then a potentially complete write (that was overwritten by greater tags in the rest of the servers) is discovered and that tag is returned. If no iteration is interrupted because of qView(1), then eventually qView(3) is observed, in the worst case, when a single server remains in some intersection of Qj. Since a second round cannot be avoided in this case, we take the opportunity to propagate the largest tag observed in Qj. At the end of the second round that tag is written to at least one complete quorum and thus the reader can safely return the corresponding value.

Theorem 5.2: Algorithm CWFk implements an atomic MWMR register.
VI. Empirical Results: Simulations

We now present experimental evaluations of our algorithms, obtained by using the NS-2 network simulator.

Experimentation Platform: Our test environment consists of a set of writers, readers, and servers. We use bidirectional links between the communicating nodes, with 1Mb bandwidth, latency of 10ms, and a DropTail queue. To model asynchrony, the processes send messages after a uniformly at random delay between 0-0.3 sec. We ran NS2 in Ubuntu, on a Centrino 1.8GHz processor. The average of 5 samples per scenario provided the stated operation latencies.

We have evaluated the algorithms with majority quorums. As discussed in [7], assuming \(|S|\) servers out of which \(f\) can crash, we can construct an \((\frac{|S|}{T} - 1)\)-wise quorum system \(Q\). Each quorum \(Q\) of \(Q\) has size \(|Q| = |S| - f\). The processes are not aware of \(f\). The quorum system is generated \(a\ priori\) and is distributed to each participant. We model server failures by selecting some quorum of servers (unknown to the participants) to be correct and allowing any other server to crash. The positive time parameter \(cInt\) is used to model the failure frequency or reliability of every server \(s\). We use the positive time parameters \(rInt = 5sec\) and \(wInt = 10sec\) to model operation frequency. Readers and writers pick a uniformly at random time between \([0 \ldots rInt]\) and \([0 \ldots wInt]\), respectively, to invoke their next read (resp. write) operation.

Algorithm SFW vs. APRX-SFW: First we compare algorithms SFW and APRX-SFW. We examine a specific scenario where the number of readers is fixed at 40 and the number of writers is fixed at 20 (other scenarios can be found in [1]). By assuming a single server failure and increasing the number of servers in the system, we evaluate the two algorithms using quorum systems with different intersection degrees. In particular, we run the scenario using 10, 15, and 25 servers that, with a single failure, yield a 9-wise, 14-wise, and 24-wise quorum system respectively. Examining the latency of the two algorithms, including both communication and computation costs, provides evidence of the heavy computational burden of algorithm SFW. In particular, we obtained the following numbers for the average read latency: (i) \(|S| = 10\), SFW \(RL = 1.72s\), APRX-SFW \(RL = 1.56s\), (ii) \(|S| = 15\), SFW \(RL = 10.72s\), APRX-SFW \(RL = 1.67s\), and (iii) \(|S| = 25\), SFW \(RL = 45min\), APRX-SFW \(RL = 1.23s\). It appears that the latency of algorithm SFW grows exponentially, whereas the latency of APRX-SFW can even improve when using quorum systems with large intersection degree (due to the larger number of fast reads). The exceedingly large delay of SFW in the scenario where \(|S| = 25\), forced us to terminate the simulation prior to its completion. The results presented above were obtained by examining the log files and taking an average of the time over all the completed read operations. We then examine the number of two-round writes. A writer performs two rounds only when the predicate does not hold. Thus, counting the number of two-round writes reveals how many times the predicate does not hold for an algorithm. Below we present the number of two round writes, out of a total 900 writes, that each algorithm performed in two different scenarios: (i) \(|S| = 10\), SFW \#2comm = 545, APRX-SFW \#2comm = 593, (ii) \(|S| = 15\), SFW \#2comm = 428, APRX-SFW \#2comm = 592.

According to our theoretical findings, algorithm APRX-SFW should allow no more than \(\log |S| \cdot RR\) two-round reads or \(\log |S| \cdot WR\) two-round writes in each scenario, where \(RR\) and \(WR\) are the number of two-round reads and writes allowed by the algorithm, respectively. Our experimental results are within the theoretical upper bound, illustrating the fact that algorithm APRX-SFW implements a \(\log |S|\)-approximation relative to algorithm SFW. These scenarios demonstrate the performance benefit of using algorithm APRX-SFW over algorithm SFW.

Algorithm CWFR vs. APRX-SFW: We now proceed to compare Algorithm APRX-SFW with the new algorithm CWFR. To examine the impact of computation on the operation latency, we also compare these algorithms to algorithm SIMPLE. This is a standard two-round read and write protocol. Both read and write operations involve a query phase to discover the maximum tag in the system; then the write operation increments the maximum tag and propagates the new tag along with the value to be written to some quorum, whereas the read operation just propagates the maximum tag to some quorum. Note that algorithm SIMPLE requires insignificant computation, and thus the latency of an operation in this algorithm directly reflects four communication delays (i.e., two rounds).

To evaluate the efficiency of the algorithms we use several scenarios. For reasons of space we present only two scenarios (all scenarios can be found in [1]). The first uses a quorum system with a small intersection degree and the second uses a quorum system with a large intersection degree: (i) \(|S| = 10\), \(f = 2\), thus \(n = 4\), and (ii) \(|S| = 15\), \(f = 1\), thus \(n = 14\). In the scenarios we use 10, 20, 40 and 80 readers, combined with 10, 20, and 40 writers respectively. We observe that in all scenarios algorithms APRX-SFW and CWFR exhibit better read and sometimes better write latency than algorithm SIMPLE. This suggests that the additional computation incurred in these two algorithms does not exceed the delay associated with a second communication round. Figures 3 and 4 depict two specific scenarios that we explain further below.

Scenario 1: In this scenario we consider a system with \(|S| = 10\) servers where 2 of them may crash, resulting in a 4-wise quorum system. Using a small intersection degree none of the predicates used in algorithm APRX-SFW can be satisfied. Reads may be fast even if the predicate does not hold. Figure 3 illustrates the run where the number of writers is fixed to 20 in (a) and (b) and the number of readers is fixed to 40 in (c). Observe from Figure 3(a) that algorithm CWFR requires fewer two-round reads than APRX-SFW. For this reason, in Figure 3(b), we observe that the average read latency of CWFR is overall lower. Since the write predicate does not hold when assuming small
intersection degree, the three algorithms require all write operations to perform two rounds. The extra computation required by algorithms CWFR and APRX-SFW explains why the write latency of these algorithms is slightly higher than the write latency of algorithm SIMPLE; see Figure 3(c).

Scenario 2: In this scenario we consider a system with |S| = 15 where a single server may crash. This scenario is designed to test the performance of the algorithms when quorum systems with large intersection degree are used. The scenario yields a 14-wise quorum system and contains 15 quorums. Figure 4 depicts the results obtained for a specific run of this scenario where the number of writers is fixed to 20 in (a) and (b) and the number of readers is fixed to 40 in (c). Due to the large intersection degree, algorithm APRX-SFW allows more fast reads than CWFR (see Figure 4(a)). Consequently, as it can be seen in Figure 4(b), algorithm APRX-SFW achieves better read latency than CWFR. Moreover, from Figure 4(c) it can be observed that APRX-SFW allows some write operations to be fast and thus, its average write latency is better than in the other approaches.

It is worth mentioning that from scenarios where the intersection degree of the deployed quorum system is of “medium” size (see [1]), algorithms APRX-SFW and CWFR incur very similar operation latencies. In general, the performance of algorithm APRX-SFW is affected by both the number of writers and the intersection degree of the underlying quorum system; algorithm CWFR exhibited more stable performance in the scenarios we tested.

VII. CONCLUSIONS

We explored the feasibility of implementing multi-writer atomic registers that enable fast, single round operations. We determined that the only such previously known algorithm incorporates a decision problem that we showed to be \textit{NP-Complete}, making the algorithm not practical. We presented more practical algorithms, one of which uses a log-approximation to speed up its computation. Simulation results illustrate the advantages of our approach. We intend to explore next whether there are specialized quorum constructions that improve the logarithmic approximation factor.

REFERENCES