
Charalampos Sergiou and Vassos Vassiliou
Networks Research Laboratory
Department of Computer Science
University of Cyprus
Nicosia, Cyprus
Email: {sergiou,vasosv}@cs.ucy.ac.cy

Abstract—Studying the behavior of Wireless Sensor Networks (WSNs) is a complex task, since the effects of significant network parameters are frequently unpredictable. This, along with the fact that in most network deployments, wireless sensor nodes are densely and randomly deployed, renders the individual study of the behavior of each sensor node impractical. In this work, we attempt to analyze, model, and estimate the maximum volume of traffic that can be carried out from the sources to the sink(s) of a WSN, without the use of any congestion control algorithms. To perform our analysis we employ a macroscopic fluid dynamic model. Using this model and three fundamental traffic variables, the packet density, packet flow, and spatial packet rate, we calculate the limits of the network flows, in terms of capacity, in the absence of congestion control. Calculating these limits helps us prove a relation between incoming and outgoing flow in the bottleneck nodes that can specify the optimal point at which the network should operate without the need of congestion control algorithms.

Keywords—Wireless Sensor Networks, Congestion Control, Traffic Control, Resource Control, Flow Dynamics, Bottleneck

I. INTRODUCTION- BACKGROUND

Studying the behavior of wireless networks is not a trivial task since a lot of parameters, usually with unpredictable behavior, are involved in this process. The situation is even more complex for WSNs, where the placement of nodes is frequently dense and random, while a big number of nodes transmit data concurrently in the case of an event. Thus, a number of authors [1]-[3], instead of studying the individual attitude of each node in terms of their microscopic parameters, choose to study the data flow from sources to sinks macroscopically, using fluid or other models. The most related (and to the best of our knowledge, the only) works in the literature employing fluid dynamics models are briefly described here.

One of the very first efforts that introduced deterministic fluid models for modeling the traffic of wireless communications networks appeared in 1994 by Leung et al. [1]. In this work the authors focus on wireless telephony and provide mathematical models to help understand system dynamics and analyze the performance of these networks. Specifically, they consider a highway with multiple entrances and exists, while vehicles can be in a calling or non-calling state. In order to perform their study, they introduce a deterministic fluid model and two stochastic traffic models for wireless networks. The deterministic model ignores the behavior of individual vehicles and treats them as a continuous fluid.

Gribaudo et al. [2] claim that the behavior of large-scale WSNs is complex and difficult to analyze, thus they develop an analytical model of the behavior of WSNs, based on a fluid approach. Actually, they represent WSNs by a continuous fluid entity distributed on the network area.

Toumpis et al. [3] investigate the spatial distribution of wireless nodes that can transport a given volume of traffic in a sensor network, while requiring the minimum number of wireless nodes. In that work they show that under specific assumptions the optimal distribution of nodes induces a traffic flow identical to the electrostatic field that would exist if the sources and sinks of traffic were substituted with an appropriate distribution of electric charges. The authors introduce three macroscopic quantities: the information density function, the node density function, and the traffic flow function. Furthermore, they suggest a relation between the node density and the traffic flow which is based on the fundamental assumption that a location \((x,y)\), where the node density is \(d(x,y)\), can support any traffic flow vector with a magnitude less or equal to a bound \(|T(x,y)|_{\text{max}}\) which is proportional to the square root of the density, i.e. \(|T(x,y)| \leq |T(x,y)|_{\text{max}} = K \sqrt{d(x,y)}\).

Concerning the estimation of the maximum volume of traffic that can be carried out from the sources to the sink, this depends on the physical and MAC layers.

Specifically, Toumpis et al. [3] studied the case when a simple time division MAC protocol that consists of only three slots is employed. In this scenario they considered a grid of \(m \times m\) nodes with a source data rate of \(W\) bps. Using this placement they assumed that each node can listen to the transmission of its four nearest neighbors (or if it is located at the edge, to the two or three nearest nodes). Using these data, they calculated that the maximum traffic that is able to transverse this network is equal to \(\frac{8W}{\pi}\).

Under a similar scenario Silvester et al. [4] proved that if, instead of a time division MAC protocol, a slotted Aloha is employed, the maximum possible traffic between the nodes is \(k \times W \times m\) where \(k\) is a constant less than \(\frac{1}{4}\).

Franceschetti et al. [5] considered a contention-based environment. They proved that under the best conditions the maximum possible traffic that can be carried out in this network
from source to sink is $\Theta(W\sqrt{m})$. This can be achieved when the flows are completely disjoint and each flow consists of $\Theta(\sqrt{m})$ wireless nodes. Each node is able to carry $k_1W$ bps, where constant $k_1$ captures the effects of a node having to share the channel with competing nodes.

In this paper we attempt to analyze, estimate, and model the maximum traffic volume that can exist in a WSN, in order to operate properly, without the need of congestion or overload control algorithms, focusing at the packet level and not at the node level as in [2] and [3] do.

The contributions of this paper are as follows: (i) initially we present a macroscopic version of the conservation of information law and we prove that this law can stand in WSNs when the maximum capacity of the network is not exceeded. In this case the data flow in the network can be represented by a continuity equation as in fluid dynamics. To the best of our knowledge this is the first time that this equation is being used in the context of Wireless Sensor Networks, (ii) using the continuity equation we can prove a relation between the incoming and outgoing flow in the bottleneck nodes, which can set the limits in data flows. Using these limits we can estimate the maximum traffic volume of the network.

The rest of the paper is organized as follows. In Section II we present the system model, in Section III we present the analytical results, and finally we close with conclusions.

II. SYSTEM MODEL

Initially, we consider a number of disjoint data flows with space variable $x \in \mathbb{R}$ and time variable $t \geq 0$ (Fig. 1) moving in one direction (e.g. from source to sink). At this point we introduce three macroscopic traffic variables: (i) packet density function $\rho(x, t)$, (ii) spatial packet rate function (packet velocity) $v(x, t)$, and (iii) flow $f(x, t)$. As packet density $\rho(x, t)$, we consider the density of packets at time $t$ at point $x$ measured in packets per distance unit. The spatial packet rate $v(x, t)$ or packet velocity is the rate of packets at time $t$ at point $x$, measured in length over time unit. Instead of keeping track of the spatial rate of each packet in the network, we assign to each point in the network a spatial packet rate field $v(x, t)$. Thus, flow is the product of $\rho(x, t) \times v(x, t)$, which is the number of packets that pass from a specific point in the network at time $t$ and it is measured in packets per time unit.

We also assume that the following statements hold:

- Packets are aware of their destination nodes.
- Lost packets are retransmitted.

The fact that lost packets are retransmitted until they reach the next hop is crucial, since if this statement holds, we can safely claim that at the macroscopic level the number of packets is “conserved”.

A. Conservation of Information (Packets)

In this subsection we prove that a macroscopic version of a conservation of information law holds in WSNs, bearing in mind the assumptions stated above. Specifically, as we stated before, flow $f(x, t)$ is the product of $\rho(x, t) \times v(x, t)$.

Thus, the initial traffic density at time $t = 0$ at point $x$ is $\rho(x, 0)$. The movement of each packet can then be specified by calculating the derivative of point $x$ with respect to $t$. This movement satisfies the following first order differential equation:

$$\frac{dx}{dt} = v(x, t) \text{ with } x(0) = x_0.$$  \hspace{1cm} (1)

The solution of (1) determines the position of each packet as the flow is evolving.

Let us consider a specific route segment between nodes $n_j$ and $n_k$ which is specified by $x = a$ and $x = b$ as it appears in Fig. 2. Now, we would like to calculate the number of packets in this segment, if we know the spatial packet rate.

For this specific network instance (Fig. 2) it is possible to calculate the number of packets $P$, by integrating the traffic density:

$$P(t) = \int_a^b \rho(x, t)dx.$$  \hspace{1cm} (2)

Assuming that no packets are created or destroyed in this segment (lost packets are retransmitted), the number of packets between $x = a$ and $x = b$ can only change due to the number of packets that enter $x = a$ and the number of packets that leave from $n_k$ at $x = b$. Therefore the traffic flow $f(a, t)$ and $f(b, t)$ is not the same as the time is evolving.

The rate of change of packets with respect to time, $dP/dt$, is equal to the number of packets per time unit entering the network segment $[a, b]$ at $x = a$ minus the number of packets per time unit exiting the segment $[a, b]$ at $x = b$. In this case the packets are always moving upstream (from source to sink) and the rate of change of the number of packets per unit time, is the traffic flow at position $a$ minus the flow at position $b$, both at time $t$, according to (3).

$$\frac{dP}{dt} = f(a, t) - f(b, t).$$  \hspace{1cm} (3)

Taking the derivative of both sides of (2) with respect to time gives the following:
\[
\frac{dP}{dt} = \frac{d}{dt} \int_a^b \rho(x,t)dx. 
\] (4)

Combining (3) and (4) we have:

\[
\frac{d}{dt} \int_a^b \rho(x,t)dx = f(a,t) - f(b,t). 
\] (5)

\(f(a,t) - f(b,t)\) can be substituted by taking the partial derivative of the right hand side of (5) with respect to \(x\). Also taking the integral from \(x = a\) to \(x = b\), and assuming that \(f(x,t)\) is smooth as a function of \(x\) and \(\rho(x,t)\) is smooth as a function of \(t\), we can apply the fundamental theorem of calculus which gives the following equation:

\[
\int_a^b \frac{\partial}{\partial t} \rho(x,t)dx = \int_a^b -\frac{\partial f(x,t)}{\partial x} dx, 
\] (6)

which can also be written as:

\[
\int_a^b \left[ \frac{\partial}{\partial t} \rho(x,t) + \frac{\partial f(x,t)}{\partial x} \right] dx = 0. 
\] (7)

If we assume that the integrand is piecewise continuous, this can only hold for all route segments between by \(x = a\) and \(x = b\), if the integrand itself is zero. This gives us:

\[
\frac{d\rho(x,t)}{dt} + \frac{\partial f(x,t)}{\partial x} = 0. 
\] (8)

From (8) we get

\[
\frac{d\rho}{dt} + \frac{\partial f}{\partial x} = 0. 
\] (9)

**Equation (9) is a partial differential equation expressing the “conservation of information”**. This continuity equation states that the information (number of packets) in a wireless sensor network can be conserved, if the number of packets that enter the network do not exceed its capacity. Using this equation we can estimate the maximum traffic volume that can exist in a network without the need of congestion control.

### III. Analytical Results

In this section we initially present a relation between packet density and traffic flow and then we study the case where flows merge in the network and create traffic bottlenecks.

**A. Relation between packet density and traffic flow**

At this point it is important to identify how the traffic flow rate varies in an established data flow when the packet density changes. In its turn, the packet density changes when the source reporting rate varies. To specify this relation we assume that the flow is a function of density and if \(\rho \in [0, 1]\) then

\[
f(\rho) = M \times \rho(1 - \frac{1}{k}\rho), 
\] (10)

where \(M\) is a normalizing coefficient measured in bps that accounts for the MAC protocol characteristics, and \(k\) is a coefficient that captures the effects of a node having to share the channel with competing nodes. The interpretation of this assumption is that, if there are no other packets in the network, then a packet can travel with no delay in the intermediate node queues, using the shortest route. This fact maximizes the data flow. On the other hand, if the packet density is at its maximum, meaning that all buffers are full, then we can safely state that the data flow is approaching zero. The accuracy of this assumption depends on the physical layer and the MAC protocol used by the network.

This assumption follows a similar observation made in the *Event to Sink Reliable Transport (ESRT)* algorithm [6]. In this paper, the authors plot the reporting frequency of source nodes vs the reliability, which is defined as the number of packets received by the sink. Note that when the reporting frequency of the nodes increases, the number of packets that are injected in the network also increases. Fig. 3 shows that the number of packets received by the sink (reliability) rises to a peak point and then it decreases as the reporting frequency becomes larger.

![Fig. 3. Normalized reliability vs. reporting frequency [6].](image)

Assuming that this relation provided in the ESRT algorithm holds for any node in the network and not just the sink, we can state that as the density of packets around a specific node increases (and this node has to forward these packets), then the flow (the number of packets that must be forwarded through this node in a specific time segment) reaches a maximum point and then decreases.

**B. Traffic Bottlenecks**

We consider two space variables \(x, y \in \mathbb{R}\), while we study a contention-based scenario. Specifically, we use the results provided by [5] and we study the case when separate flows merge at a specific node in the network (Fig. 4).

![Fig. 4. Separate flows merge at a single node.](image)

According to [5], each node is able to carry \(kW\) bps, where...
constant $k$ captures the effects of a node having to share the channel with competing nodes.

Initially we consider $n$ active flows, which come from an equal number of nodes, and which cross on a specific node (the bottleneck). All other nodes in the network are in a sleep state and do not interfere with the active nodes of the network. All of the active nodes share the access medium and each one is, ideally, able to transmit $\frac{W}{n}$ bps, meaning that $k$ is equal to $\frac{1}{n}$. It is clear that if the $n$ flows do not take this restriction into account and continue to transmit packets at the full rate ($W$ bps), then buffer-based congestion is going to appear at the nodes before the common node. These nodes will receive $n$ times the number of packets they are able to forward, unless the value of $\rho$ is calculated and used in reducing the source sending rate.

To calculate this value we must prove a relation between incoming and outgoing flows so as to avoid queue formation in the bottleneck node and subsequently to all backward nodes. Thus, we attempt to define the proper density of packets that the initial flow should have, in order to avoid the formation of queues at the bottleneck node.

We start by defining the initial conditions of the flows before $(f_1)$ and after $(f_2)$ the bottleneck node. Specifically, before the bottleneck node, we assume that according to (10) with coefficients $M = 1$ and $k = 1$ ($n = 1$), the flow function of each active flow is:

$$f_1(\rho_{1,0}) = \rho_{1,0}(1 - \rho_{1,0}), \rho_{1,0} \in [0, 1],$$  

while after the bottleneck node (in the congested part) the flow function is

$$f_2(\rho_{2,0}) = \rho_{2,0}(1 - \frac{1}{k}\rho_{2,0}), \rho_{2,0} \in [0, 1] \text{ and } k = \frac{1}{n}. \quad (12)$$

As a general example we provide an analysis when the number of flows $n = 2$, i.e. two different flows merge at the common node. Plotting (11) and (12), for $k = \frac{1}{n} = \frac{1}{2}$, we get Fig. 5.

Since the flow is restricted after the bottleneck, we must calculate the maximum flow that is able to pass through the bottleneck. Studying Fig. 5, it is clear that it is enough to calculate the maximum point of the curve that represents the restricted flow. Thus, we calculate the points where the first derivative of the function $f_2(\rho_{2,0})$ turns to zero, as follows:

$$f_2'(\rho_{2,0}) = 0 \Rightarrow [\rho_{2,0}(1 - \frac{1}{k}\rho_{2,0})]' = 0 \Rightarrow \rho_{2,0} = \frac{k}{2}. \quad (13)$$

The implication for the case analyzed here is that in order to avoid queue formation, the flow entering the bottleneck node should be limited to $f(\rho) = \rho(1 - \frac{1}{k}\rho)$ where $\rho = \frac{k}{2}$. The result is that $f_1(\rho)$ should not be greater than $\frac{k}{4}$.

To calculate the point where queue formation begins (for the nodes before the bottleneck) we solve the following equation with respect to the density $\rho$.

$$f(\rho) = \rho(1 - \rho) = \frac{k}{4} \Leftrightarrow \bar{\rho} = \frac{1 - \sqrt{1 - k}}{2}. \quad (14)$$

Therefore, if we do not want any queue formation, the boundary condition on the initial flow should be $f_1(\rho_{1,0}) < f(\bar{\rho})$.

Finding this point in Fig. 5, for $k = 1/2$, we notice that queue formation starts very early (when $\rho$ is 0.146 and $f$ is 0.125) which is well before the density value which provides the maximum flow at the bottleneck node.

### IV. Conclusion and Future Work

In this work we prove that a macroscopic version of a conservation law can hold in WSNs. Using this law we prove a relation between the incoming and the outgoing flow in the bottleneck nodes that can specify the maximum traffic volume of a WSN, in order to operate properly without the need of congestion control algorithms. Specifically, we prove that in case of a bottleneck in the network the incoming data flow rate must be reduced in accordance with (14) in order to avoid queue formation.

As it is proven from [5], the maximum possible traffic that can be achieved in a contention-based wireless network is achieved only when flows do not cross between them. WSNs, due to the plethora of nodes that are deployed in a field, are an ideal case for achieving disjoint flows, using alternative path creation routing mechanisms.

### References