Inverse Kinematics Solutions using Conformal Geometric Algebra

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Table of contents

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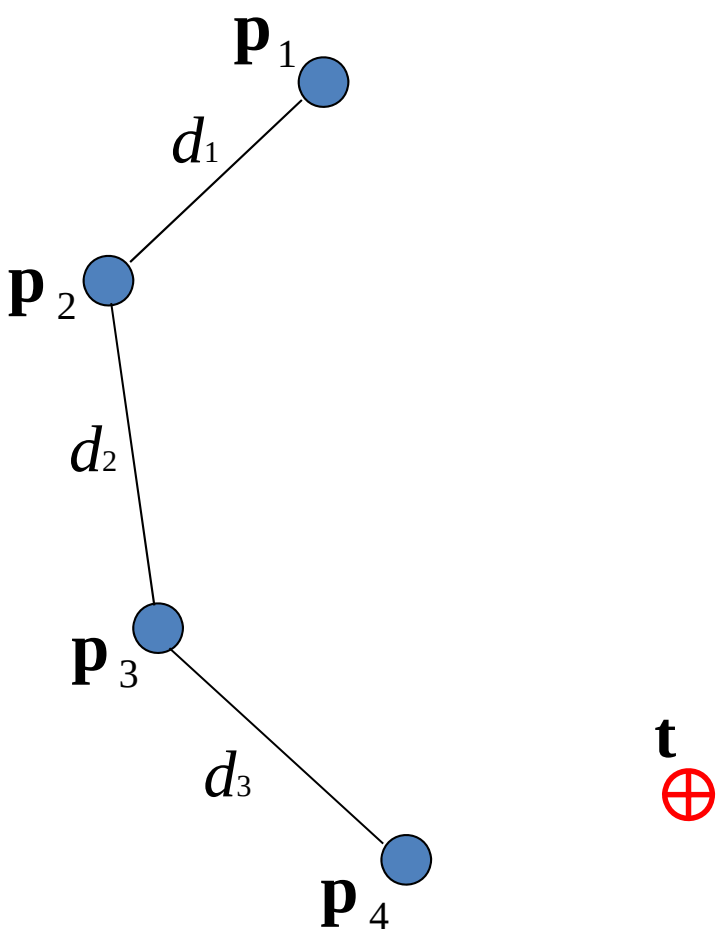
FABRIK: The Algorithm

Forward Reaching Step



The initial position of the manipulator and the target.

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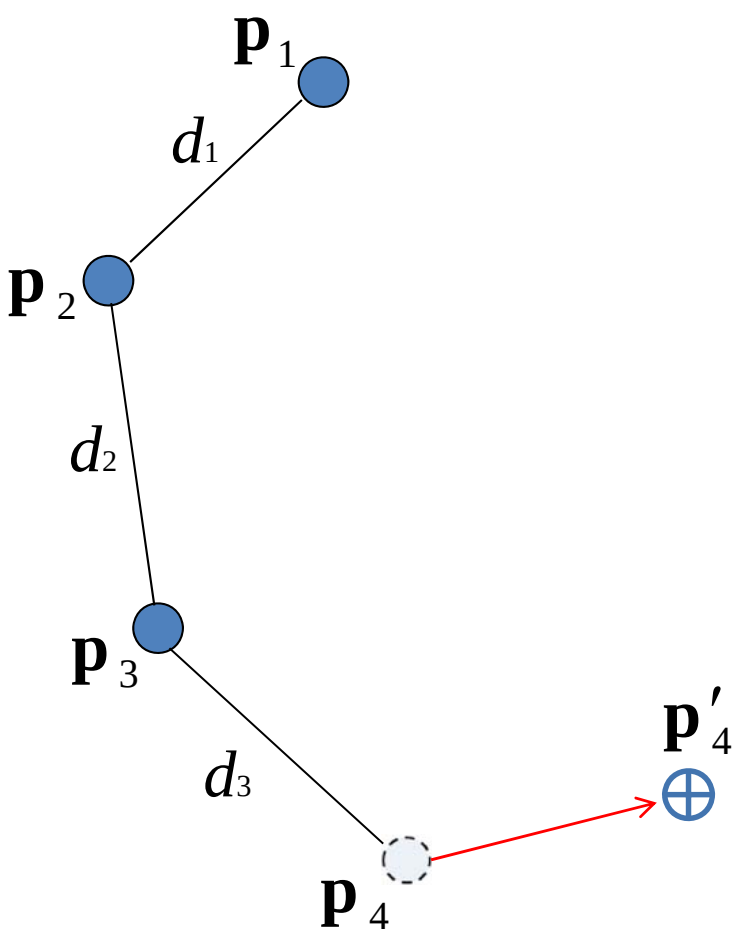

Move the end effector $\mathbf{p}_4$ to the target.

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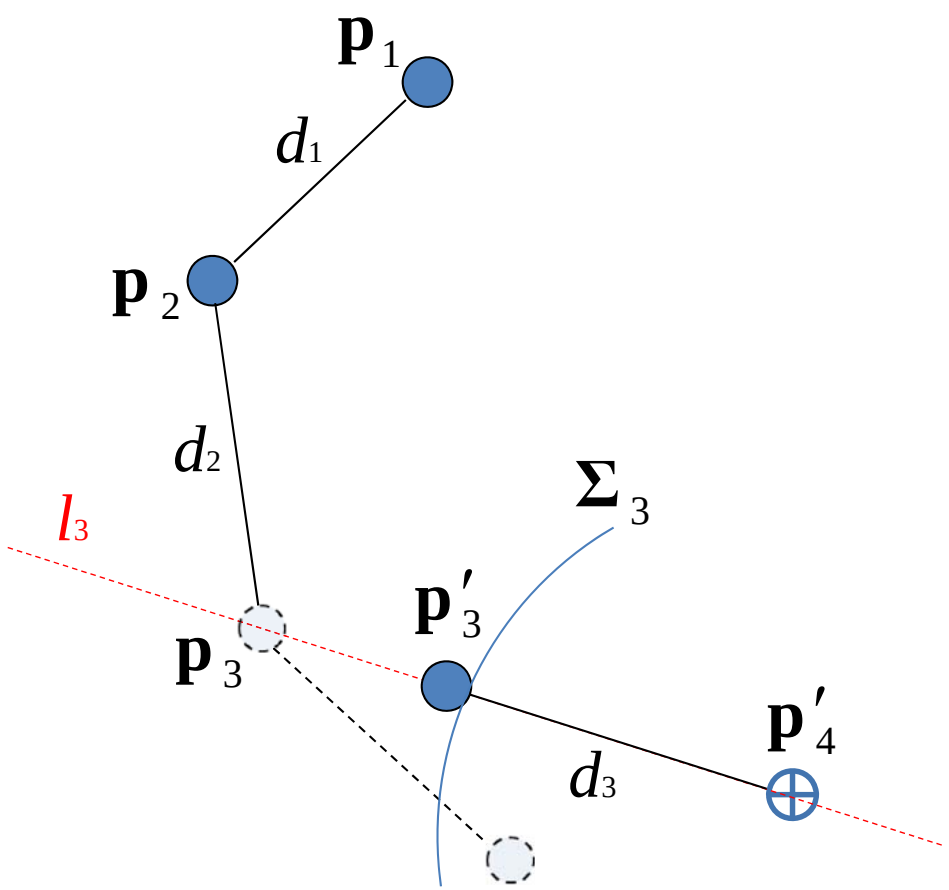

Find the joint $\mathbf{p}'_3$ which is the intersection of the sphere Σ_3 and the line l_3 which passes through the points $\mathbf{p}'_4$ and $\mathbf{p}_3$.

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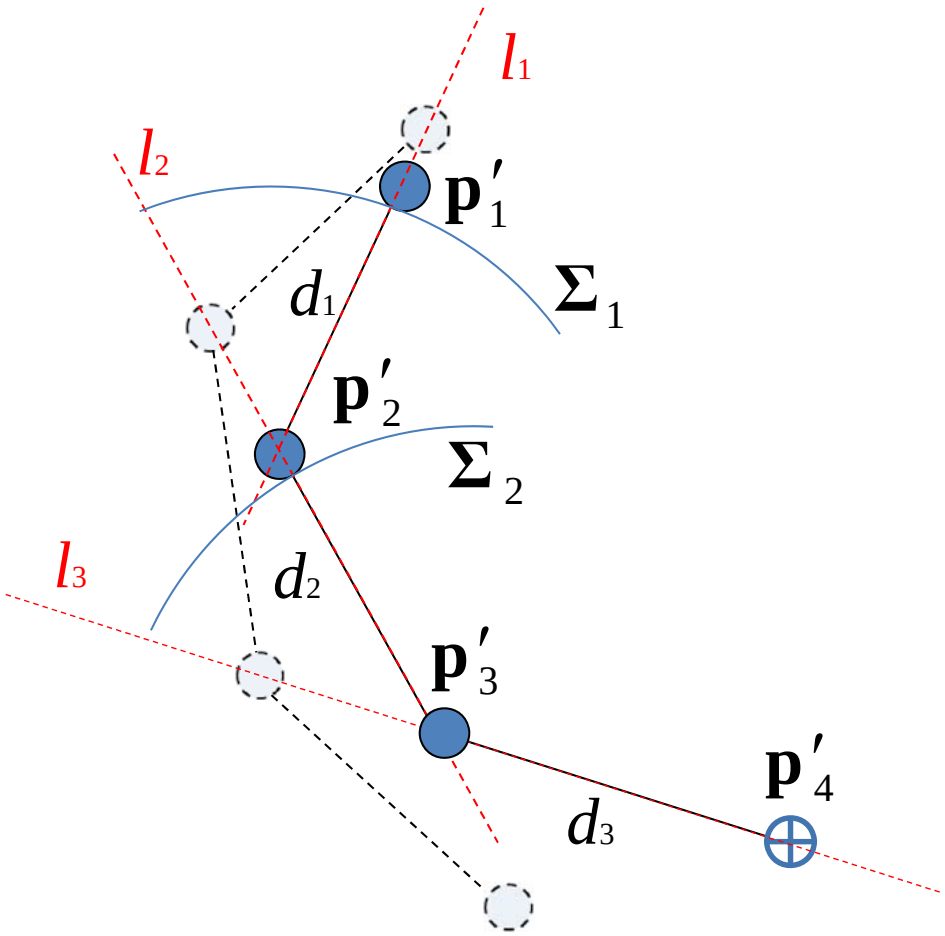

Continue the algorithm for the rest of the joints.

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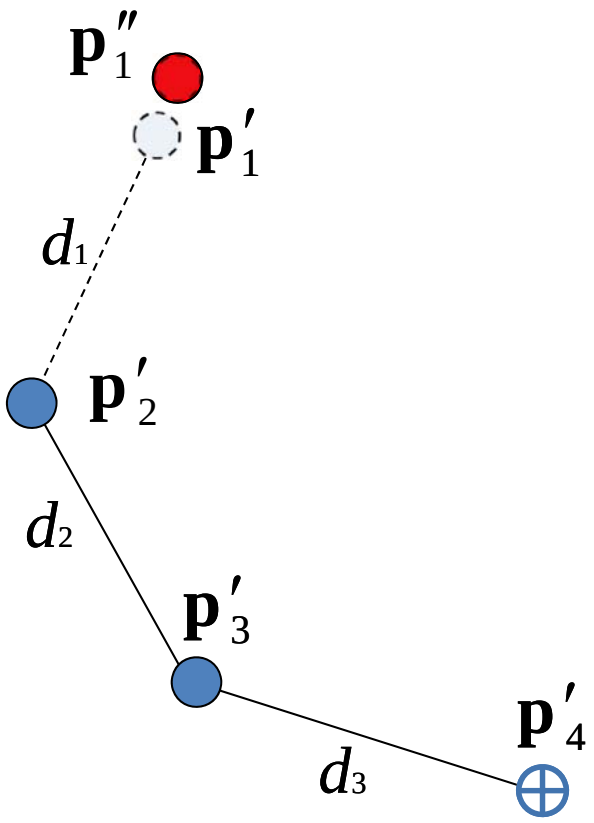
Backward Reaching Step



The second stage of the algorithm: move the root joint $\mathbf{p}'_1$ to its initial position.

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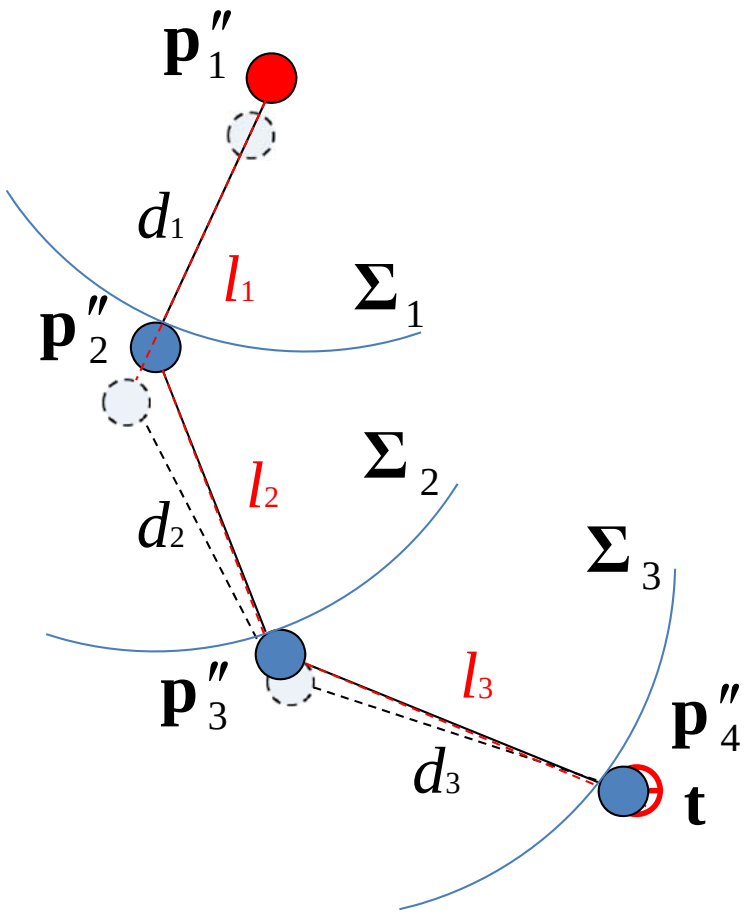

Repeat the same procedure but this time start from the base and move outwards to the end effector.

The algorithm is repeated until the position of the end effector reaches the target or gets sufficiently close.

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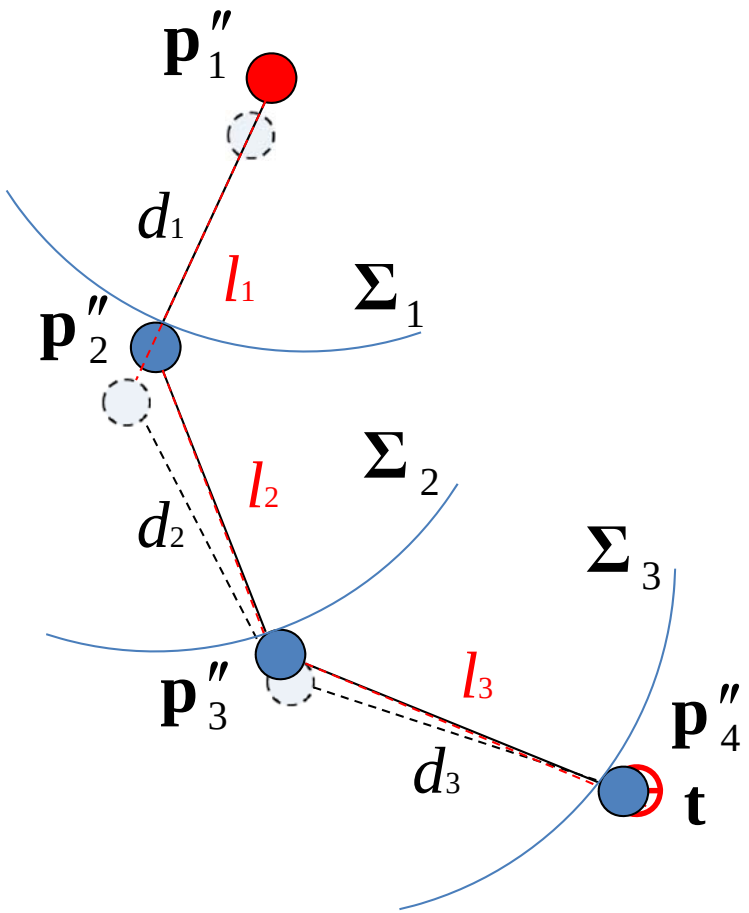

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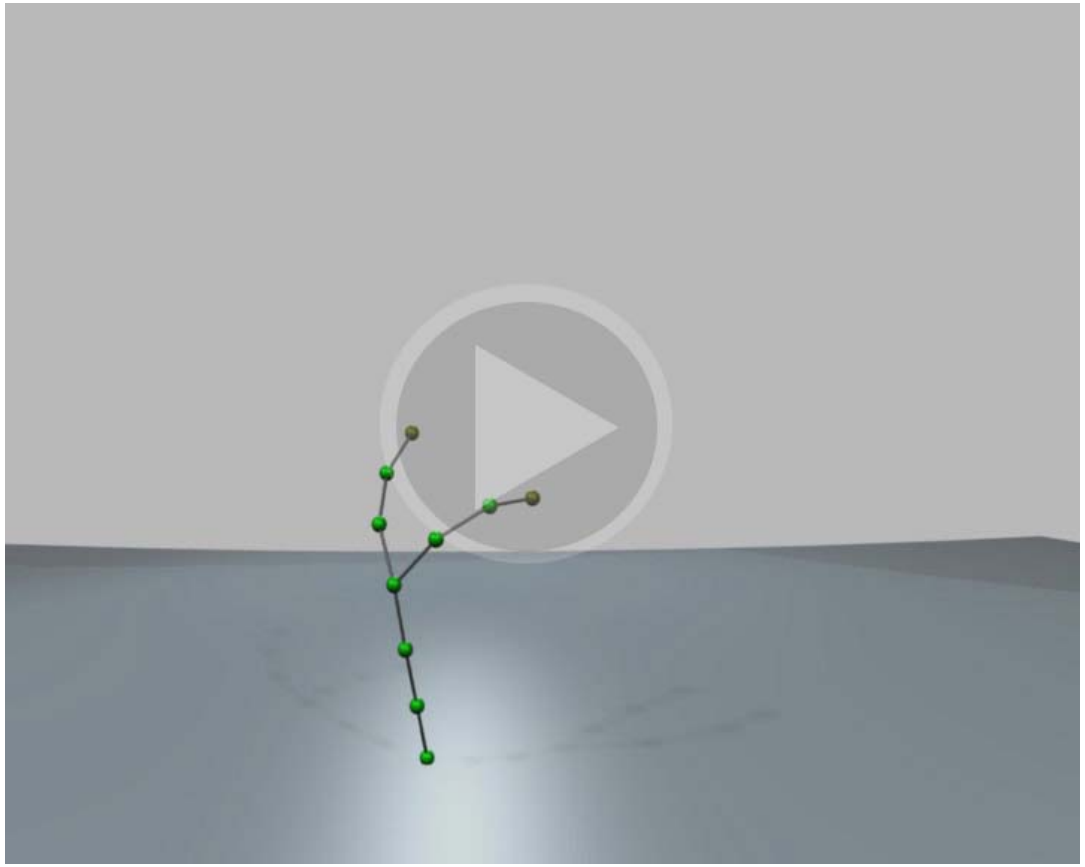
FABRIK video example



Hand tracking in optical motion capture

An application of the proposed algorithm that demonstrates all of FABRIK's advantages.

- Hand Tracking and reconstruction using optical motion capture,

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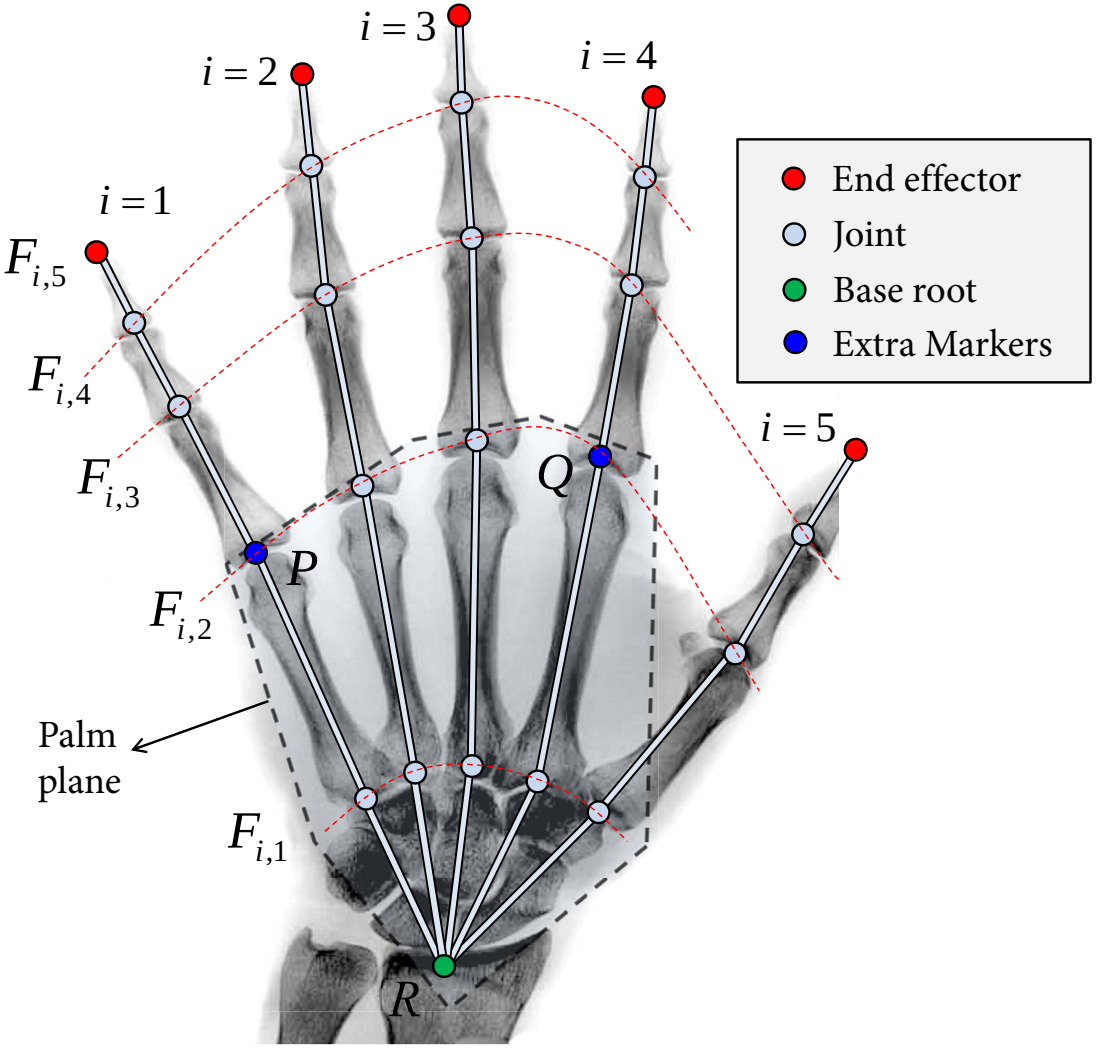

- 5 markers are attached at the end of each finger (end effectors),
- 1 marker is attached at the hand root,

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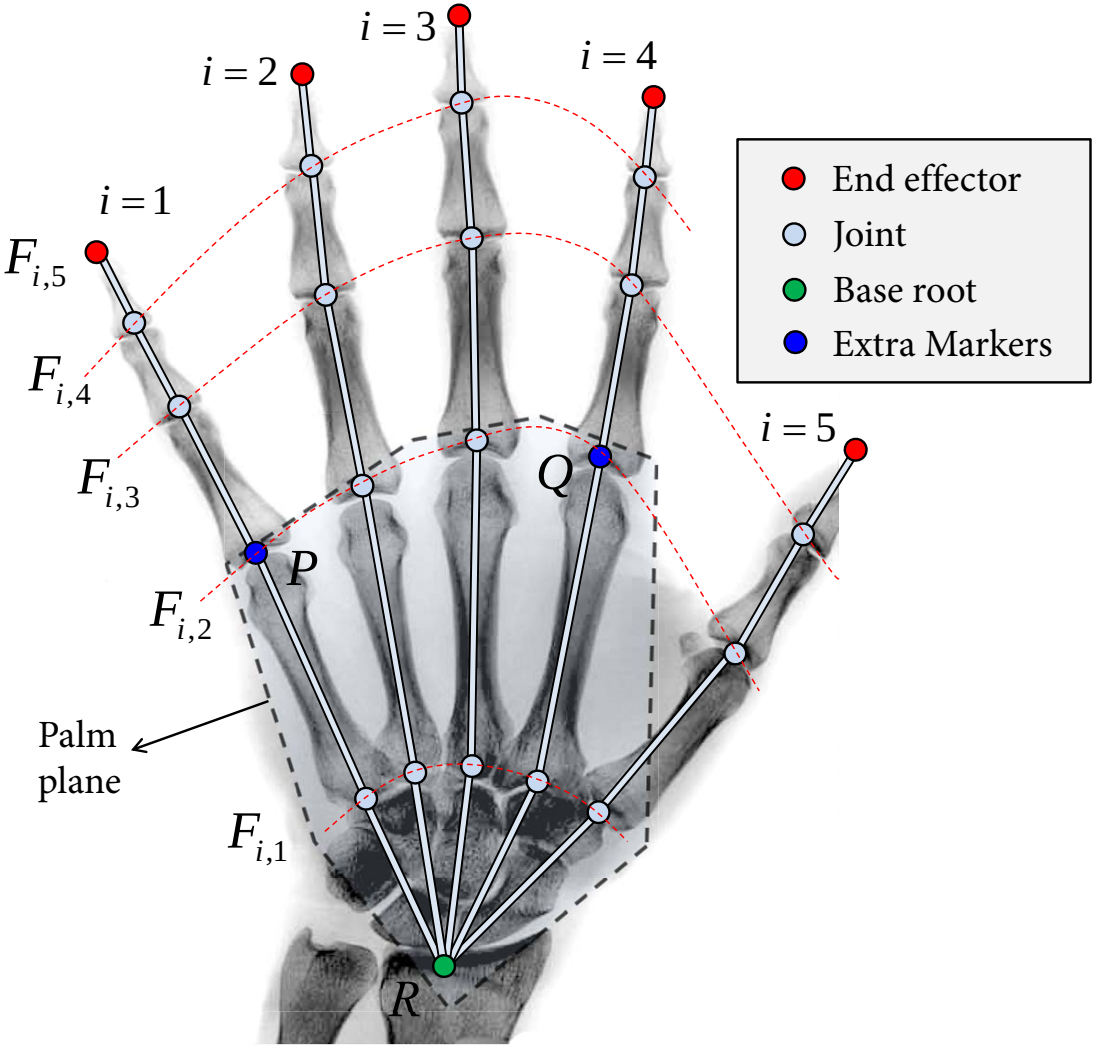

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- 2 markers are attached at strategic points on the palm to help us find the hand orientation.

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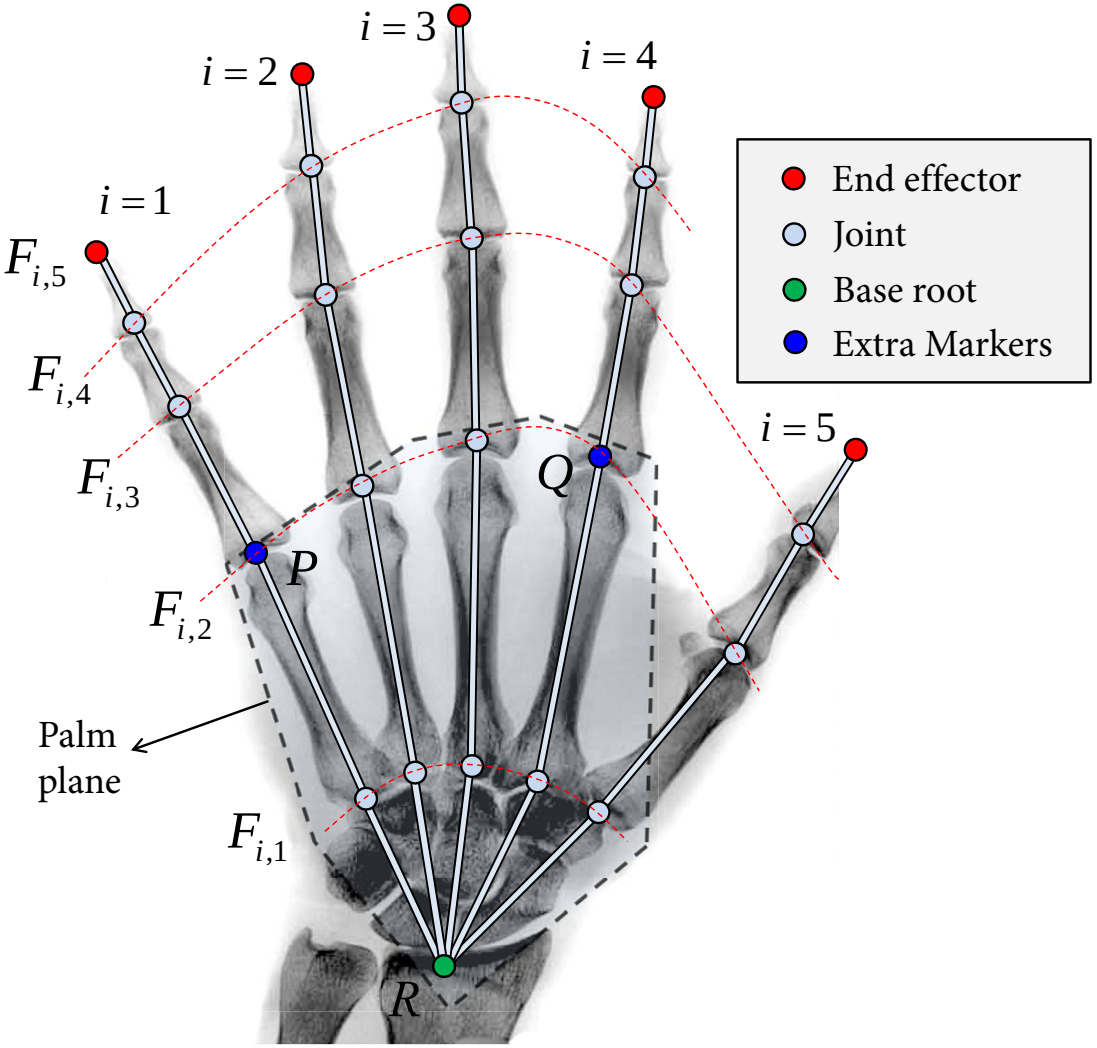

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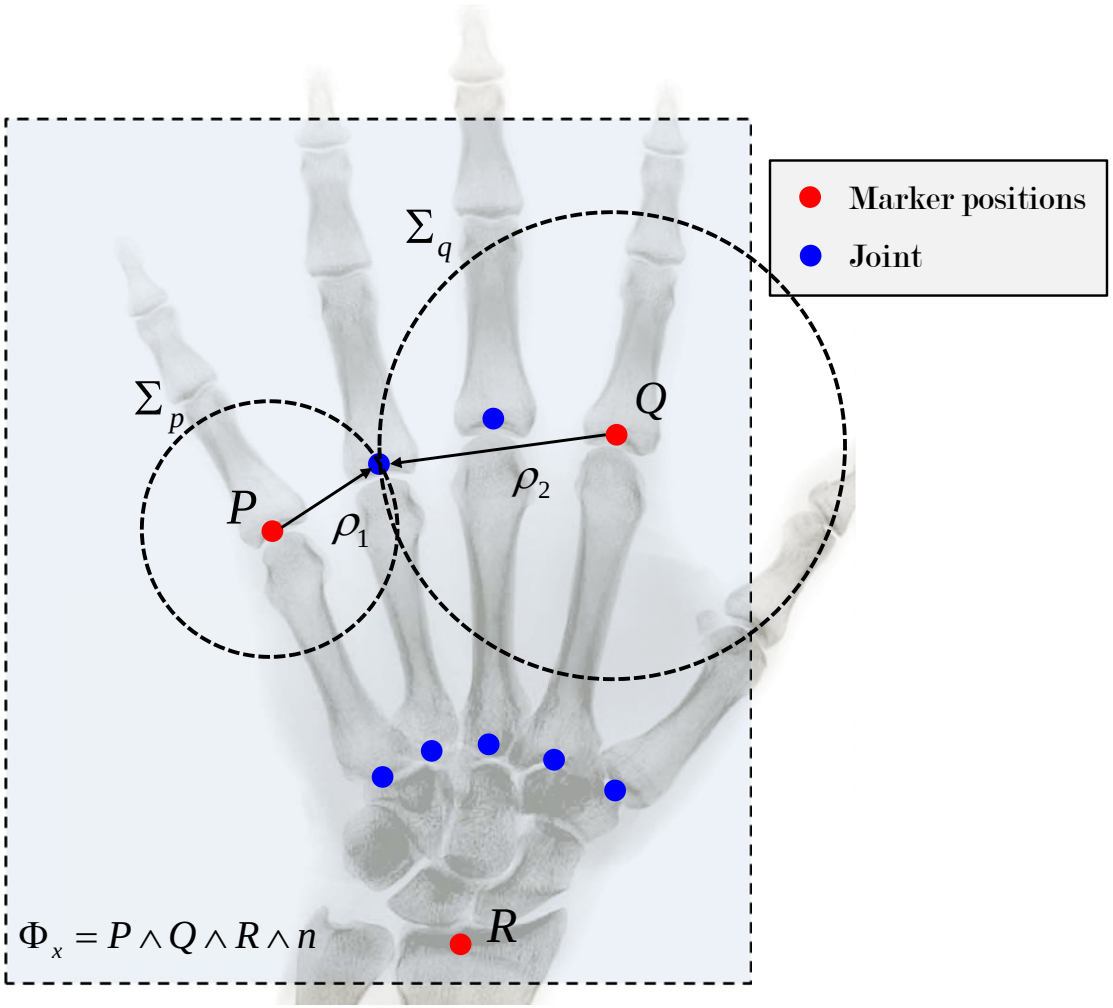
Calculating the palm joints



- Find the Spheres Σ_p and Σ_q,
- Find the intersection of the spheres $C = \Sigma_p \vee \Sigma_q$,

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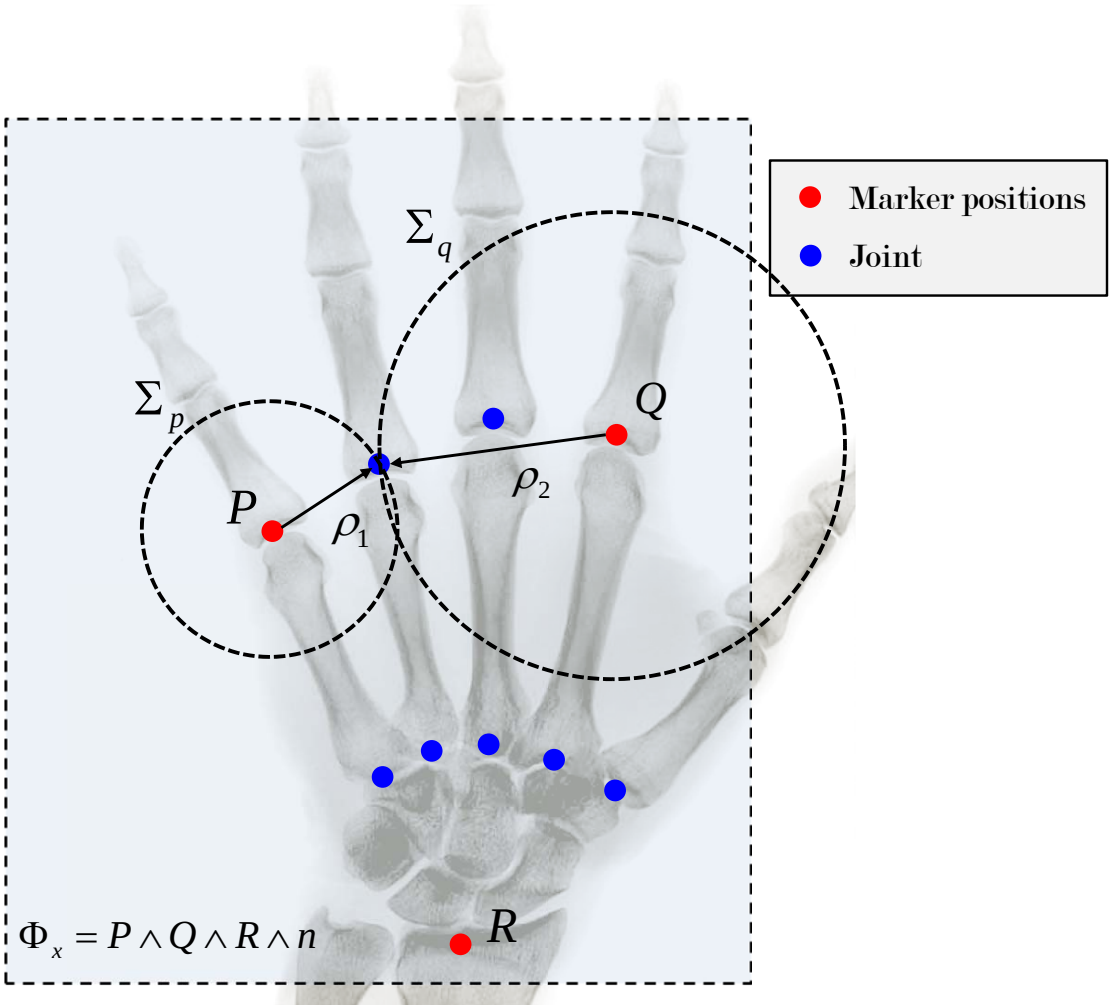

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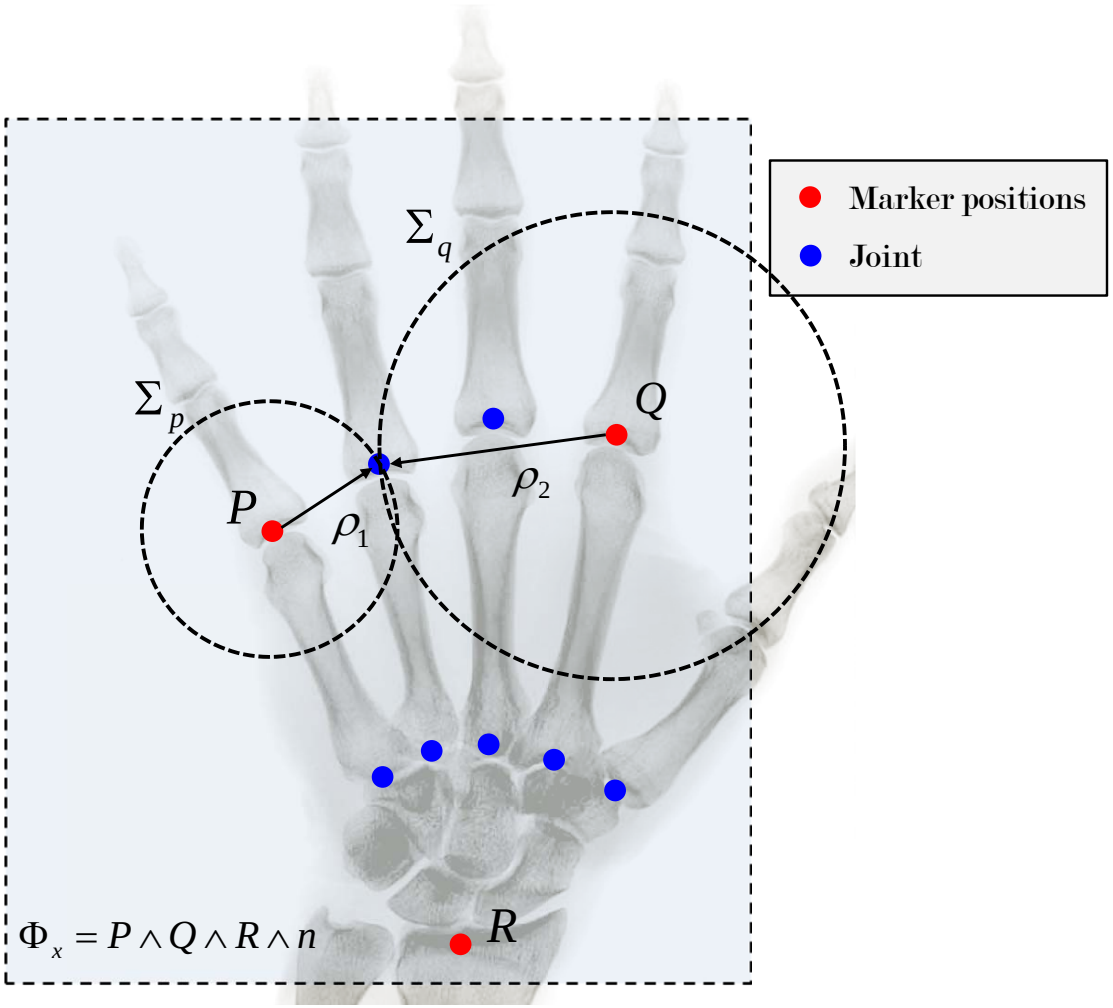

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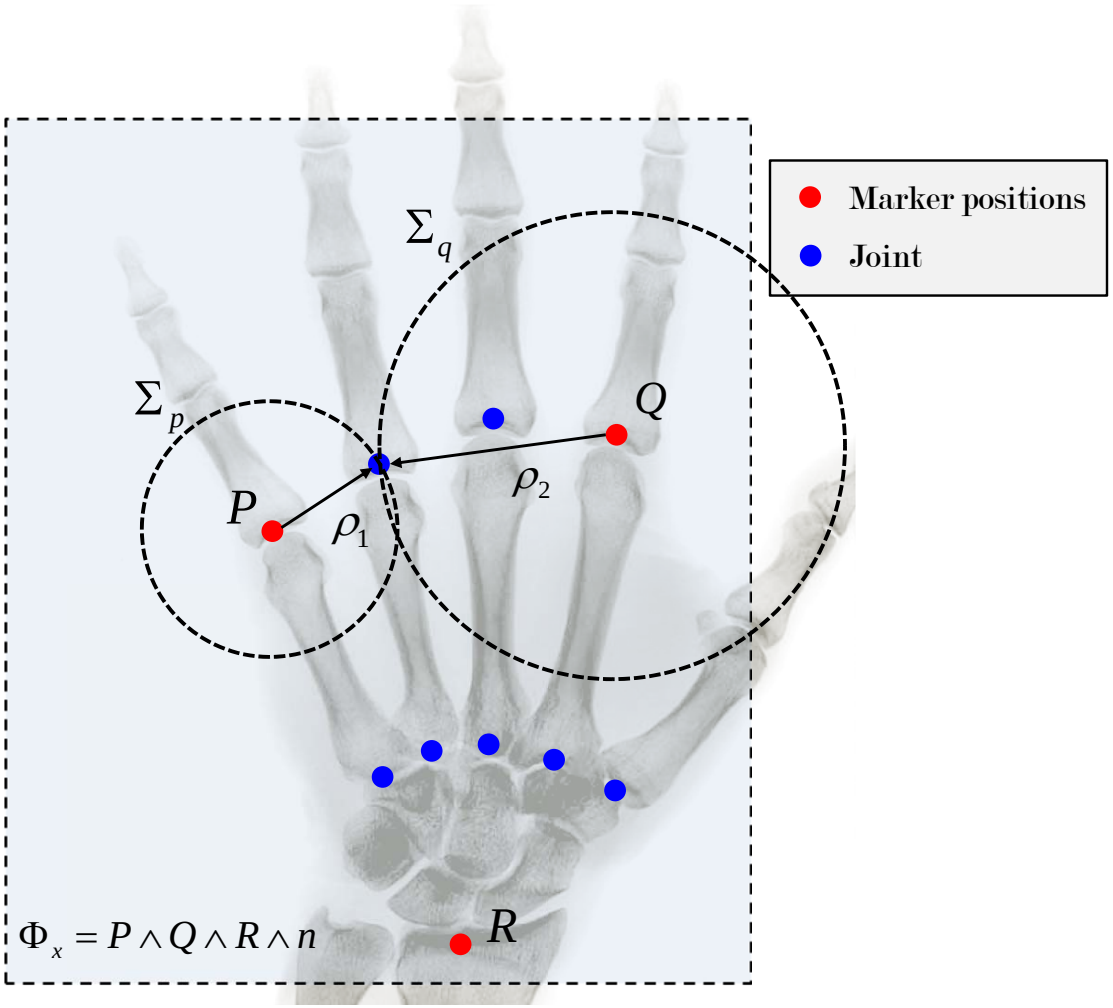

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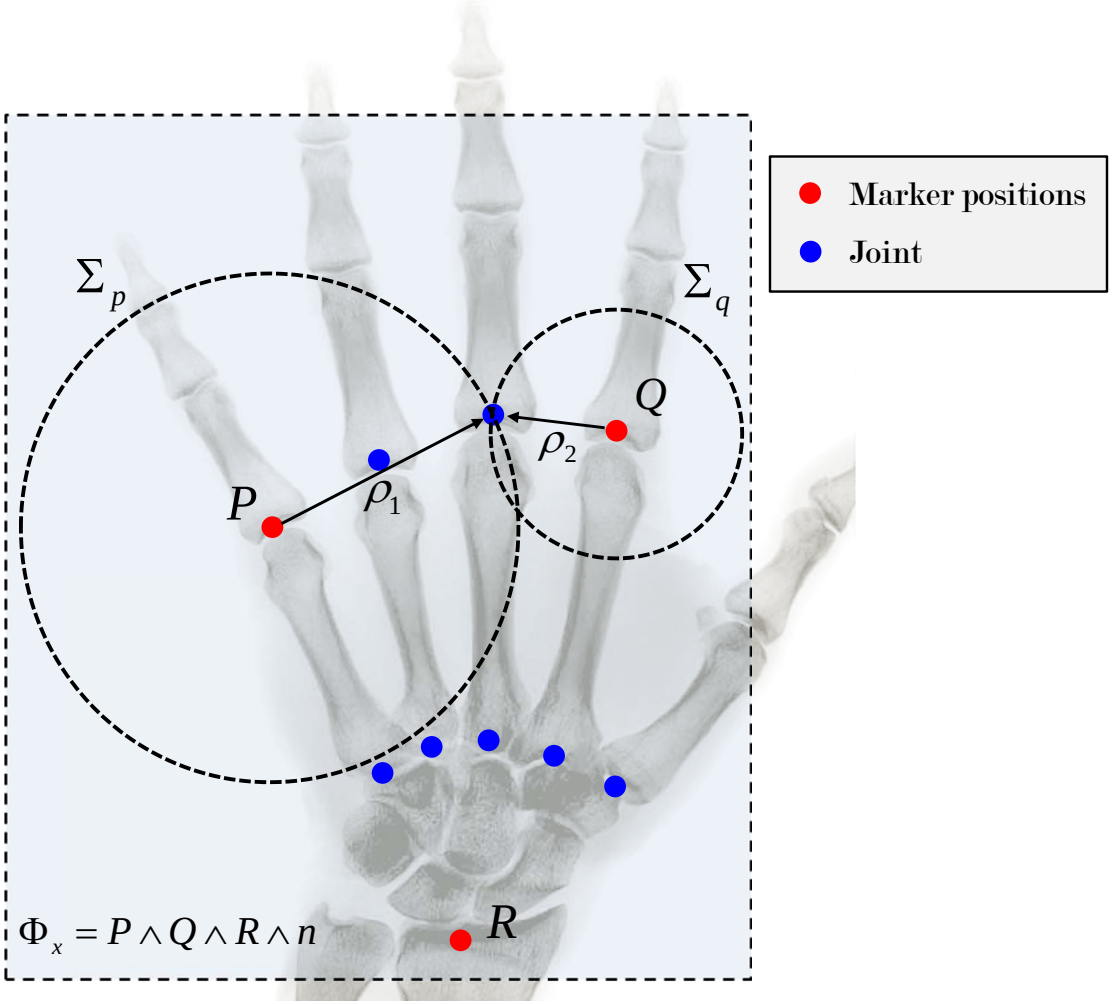

- Similarly for all the remaining palm joints

Calculating the finger joints

Find the finger planes Φ_i, for $i = 1, \ldots, 4$

- Φ_i can be calculated using the known joint positions $F_{i,2}$, the marker positions $F_{i,5}$ and by assuming that the finger planes are perpendicular to the palm plane Φ_x (apart from the thumb plane Φ_5),

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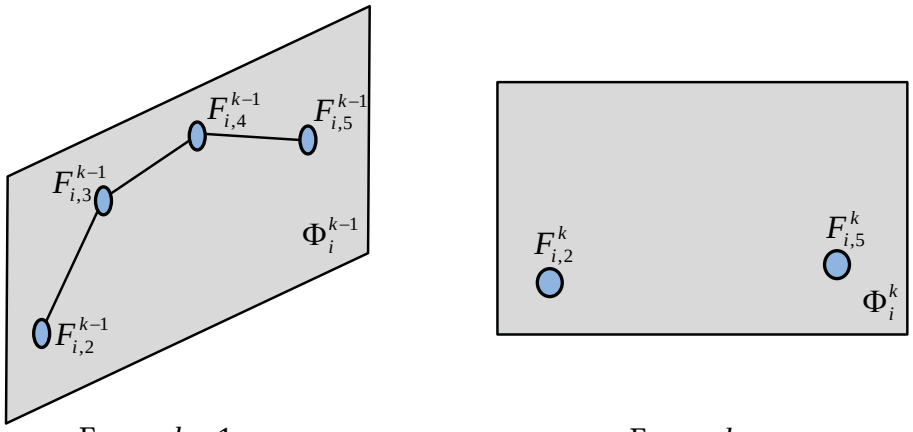
Calculating the finger joints

- The thumb orientation Φ_5 can be estimated using the marker position $F_{5,4}$, and the joints positions $F_{1,2}$ and $F_{5,2}$ that lie on the palm, assuming that when the thumb bends to the ventral side of the palm, it always points at the joint $F_{1,2}$.

Calculating the finger joints

When the finger planes are known



Frame $k-1$ Frame k

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Calculating the finger joints

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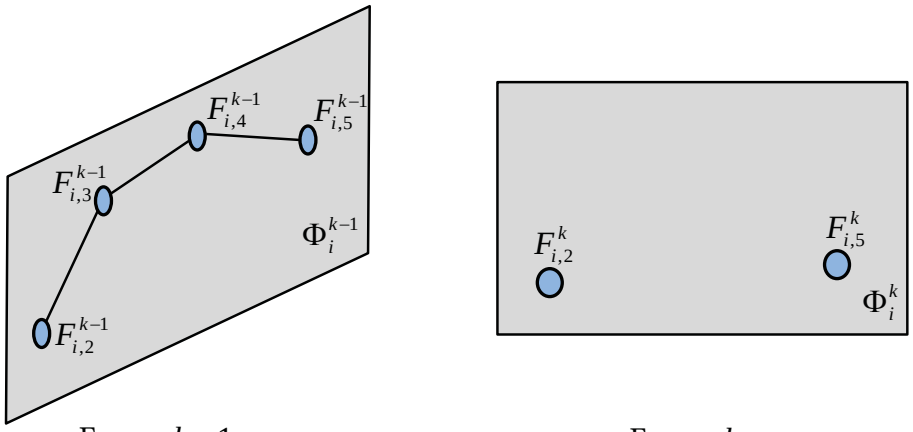

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Calculating the finger joints

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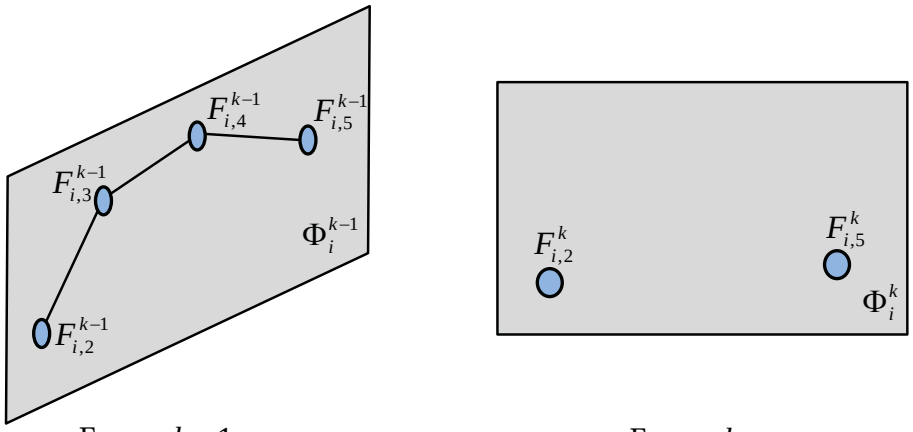

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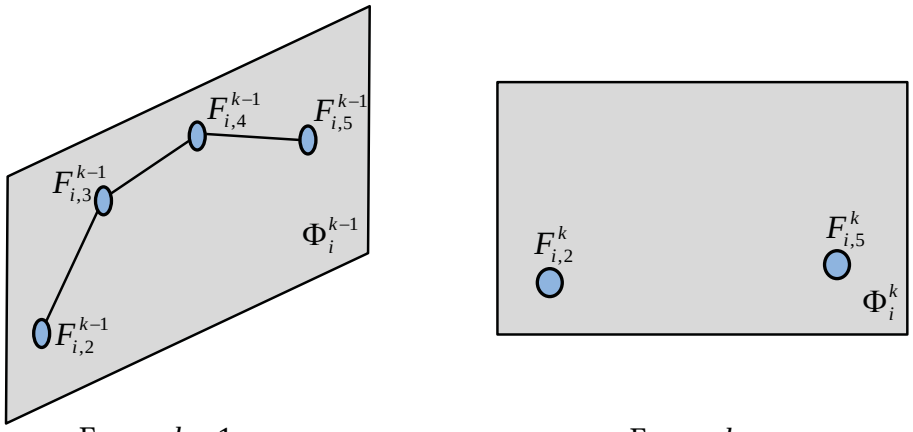

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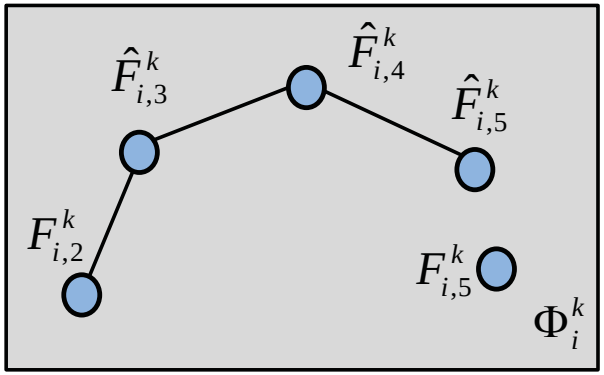
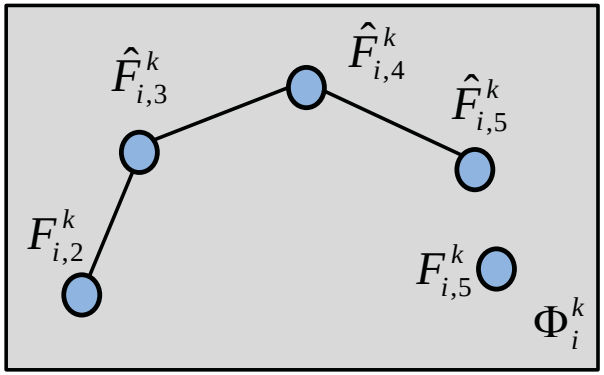


Frame k

- $\hat{F}^k_{i,j} = TRF^{k-1}_{i,j}\tilde{R}\tilde{T}$ where $i = 1, \ldots, 4$ and $j = 3, 4, 5$ (except for thumb $i = 5$ and $j = 2, 3, 4$),
- All joints now lie on plane Φ^k_i,

Calculating the finger joints



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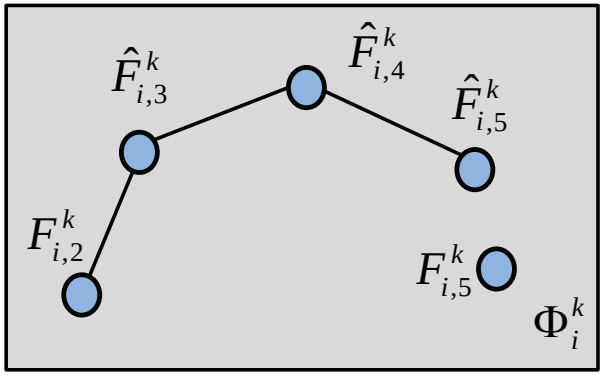
- All joints now lie on plane Φ^k_i,
- FABRIK is applied to each finger chain, assuming that the root of the chain is $F^k_{i,2}$, the end effector is the rotated point $\hat{F}^k_{i,5}$ and the target is the current marker position $F^k_{i,5}$.

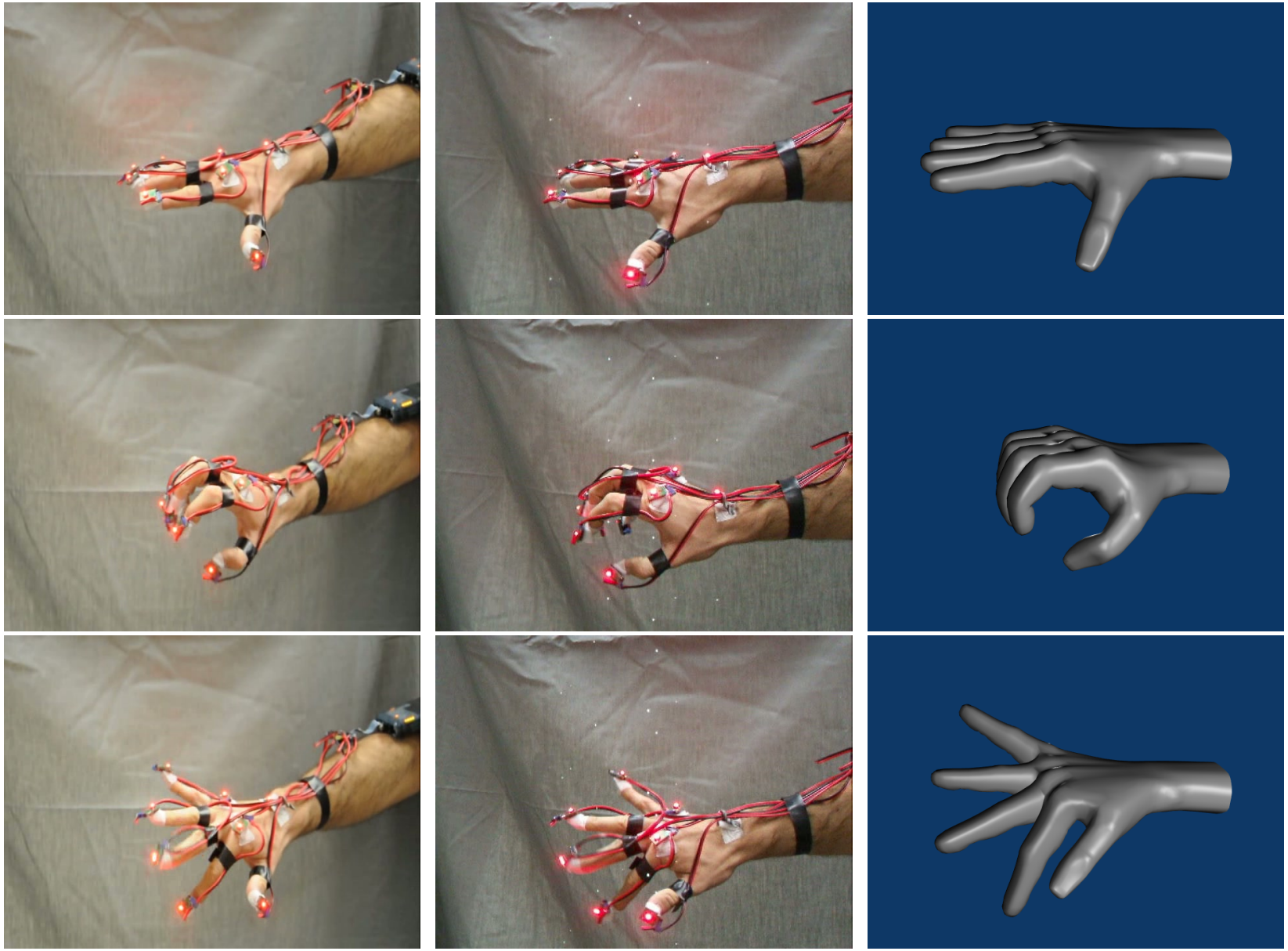
Calculating the finger joints



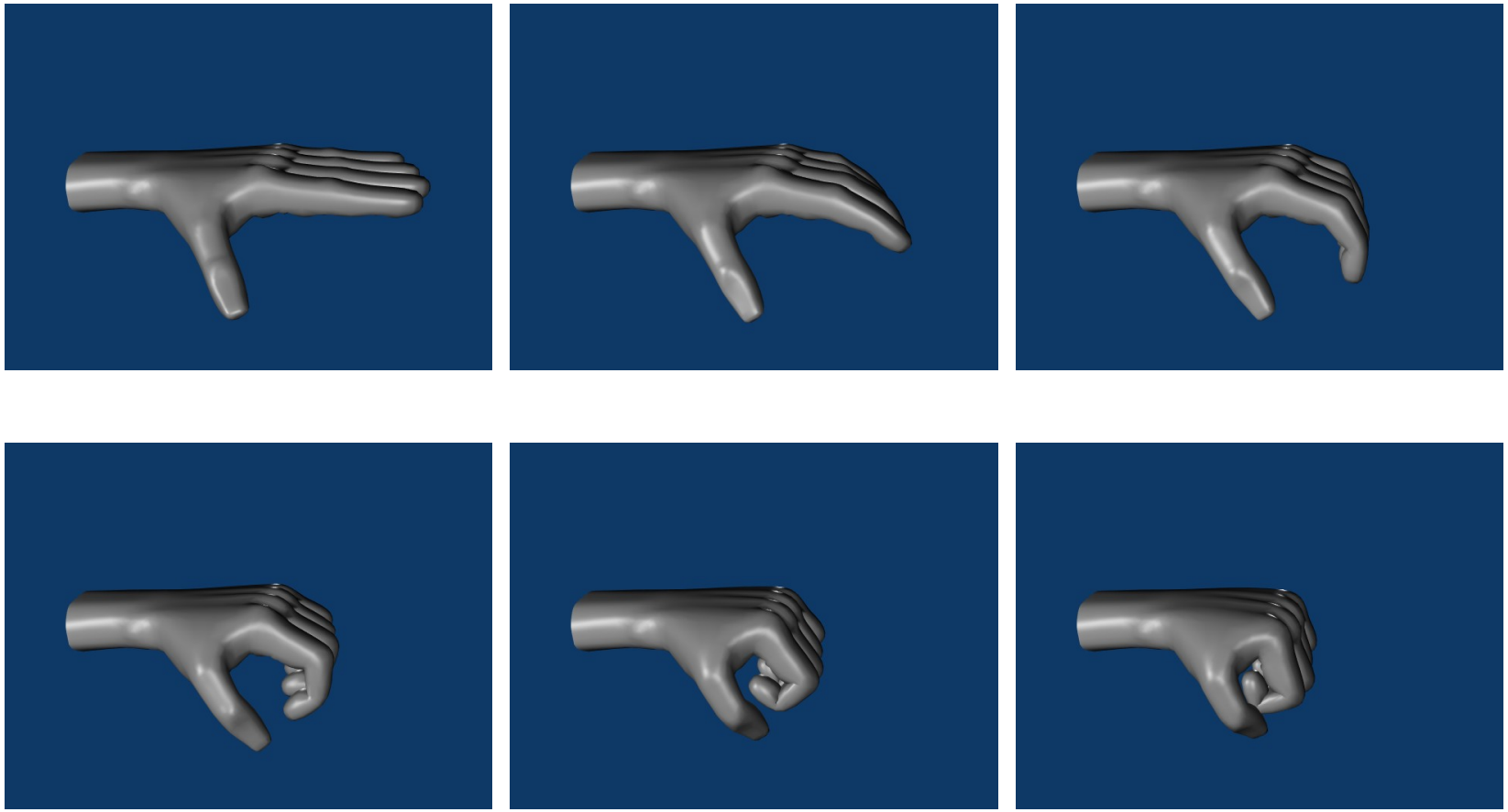
Frame k

- $\hat{F}^k_{i,j} = TRF^{k-1}_{i,j}\tilde{R}\tilde{T}$ where $i = 1, \ldots, 4$ and $j = 3, 4, 5$ (except for thumb $i = 5$ and $j = 2, 3, 4$),
- All joints now lie on plane Φ^k_i,
- FABRIK is applied to each finger chain, assuming that the root of the chain is $F^k_{i,2}$, the end effector is the rotated point $\hat{F}^k_{i,5}$ and the target is the current marker position $F^k_{i,5}$.

Results - Example 1



Results - Example 2



Results - Video

Conclusions

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Future Work

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Questions

Thank you! Any questions?

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