Preference-based Argumentation

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Outline

1. General Argumentation
2. Preference-based Argumentation (PBA)
3. Structural properties of PBA
4. Computational properties of PBA
5. PBA and Negotiation
What is Argumentation?

Argumentation = a reasoning model based on the construction, exchange and evaluation of arguments
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- **Argument** = a *reason / justification* for some **claim**
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- The core of an argument: Reasons + a claim
  - **Reason**: Because Tweety is a bird and birds fly
  - **Claim**: Tweety flies
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- **Argument** = a **reason / justification** for some **claim**

- The core of an argument: Reasons + a claim
  - **Reason**: Because Tweety is a bird and birds fly
  - **Claim**: Tweety flies

- Argumentation can be used for:
  - Internal agent’s reasoning
  - Modelling interactions between agents
What is an Argument?

- A set of premises in support of a conclusion/claim
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  - **claim**: Info $I$ about John should be published

    because
What is an Argument?

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**claim**: Info $I$ about John should be published

**because**

**premise/reason**:

John has political responsibilities

and

$I$ is in the national interest

and

if a person has pol. resp. and info about that person is in the national interest then that info should be published
What is Argumentation?

- The process of argument construction, exchange and evaluation in light of their interactions with other arguments
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- A1 (publish info about John because he has responsibilities...)

- A2 (John does not have pol. resp. because he resigned from parliament, and if a person resigns...)

Y. Dimopoulos (UCY)
What is Argumentation?

- The process of argument construction, exchange and evaluation in light of their interactions with other arguments

- A1 (publish info about John because he has responsibilities...)

- A2 (John does not have pol. resp. because he resigned from parliament, and if a person resigns...)

- A3 (John does have pol. resp. because he is now middle east envoy, and if a person...)
Δ is a set of propositional logic formulae

\[ \text{Args} = \{ (H, h) | H \subseteq \Delta \text{ is consistent}, H \vdash h, H \text{ is minimal} \} \]
$\Delta$ is a set of propositional logic formulae

$\text{Args} = \{(H, h) | H \subseteq \Delta \text{ is consistent} \land H \vdash h \land H \text{ is minimal}\}$

$(H_1, h_1)$ and $(H_2, h_2)$ rebut each other iff $h_1 \equiv \neg h_2$
Arguments in Propositional Logic

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- $(H_1, h_1)$ and $(H_2, h_2)$ rebut each other iff $h_1 \equiv \neg h_2$

- $(H_1, h_1)$ undercuts $(H_2, h_2)$ iff $h_1 \equiv \neg h$ for some $h \in H_2$
\[ \Delta = \{ \text{nat, pol, nat} \land \text{pol} \rightarrow \text{pub}, \quad \text{res, res} \rightarrow \neg \text{pol}, \quad \text{mid, mid} \rightarrow \text{pol} \} \]
Arguments in Propositional Logic

- $\Delta = \{nat, pol, nat \land pol \rightarrow pub, \ res, res \rightarrow \neg pol, \\ mid, mid \rightarrow pol\}$

- $A_1 = (\{nat, pol, nat \land pol \rightarrow pub\}, pub)$
Arguments in Propositional Logic

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\[ A_1 = (\{ \text{nat}, \text{pol}, \text{nat} \land \text{pol} \rightarrow \text{pub} \}, \text{pub}) \]

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\[ A_2 \rightsquigarrow A_1 \]
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Abstract argumentation theories (Dung 1995)

- An **argumentation theory** is a pair $\langle A, R \rangle$ where:
  - $A$ is a set of arguments
  - $R \subseteq A \times A$ is an attack relation between arguments
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- **Example**
  - Usually, Quakers are pacifists
  - Usually, Republicans are not pacifists
  - Nixon is both a Quaker and a Republican
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Example

- Usually, Quakers are pacifists
- Usually, Republicans are not pacifists
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\[ \Rightarrow \text{two arguments:} \]

- \( a : \) Nixon is a pacifist since he is a Quaker
- \( b : \) Nixon is not a pacifist since he is a Republican
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$\implies$ two arguments:
- $a :$ Nixon is a pacifist since he is a Quaker
- $b :$ Nixon is not a pacifist since he is a Republican

$a \iff b$
Abstract argumentation theories

Which arguments to accept together? \(\implies\) acceptability semantics
Abstract argumentation theories

- Which arguments to accept together? $\Rightarrow$ acceptability semantics

- Let $\mathcal{B} \subseteq \mathcal{A}$.
  - $\mathcal{B}$ is conflict-free iff $\nexists a, b \in \mathcal{B}$ such that $(a, b) \in \mathcal{R}$
Abstract argumentation theories

- Which arguments to accept together? $\implies$ acceptability semantics

- Let $\mathcal{B} \subseteq \mathcal{A}$.
  - $\mathcal{B}$ is conflict-free iff $\not\exists a, b \in \mathcal{B}$ such that $(a, b) \in \mathcal{R}$
  - $\mathcal{B}$ defends an argument $a$ iff $\forall b \in \mathcal{A}$, if $(b, a) \in \mathcal{R}$, then $\exists c \in \mathcal{B}$ such that $(c, b) \in \mathcal{R}$
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For instance:

\[c \implies b \implies a\]
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- For instance:
  \[
  c \rightarrow b \rightarrow a
  \]
  - The set \(\{c\}\) is conflict-free and defends \(a\)
Abstract argumentation theories

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Let \( \mathcal{B} \subseteq \mathcal{A} \).

- \( \mathcal{B} \) is conflict-free iff \( \nexists \ a, b \in \mathcal{B} \) such that \( (a, b) \in \mathcal{R} \)

- \( \mathcal{B} \) defends an argument \( a \) iff \( \forall \ b \in \mathcal{A}, \) if \( (b, a) \in \mathcal{R} \), then \( \exists \ c \in \mathcal{B} \) such that \( (c, b) \in \mathcal{R} \)

For instance:

\[ c \rightarrow b \rightarrow a \]

- The set \( \{c\} \) is conflict-free and defends \( a \)
- The sets \( \{a, b\} \), \( \{b, c\} \) and \( \{a, b, c\} \) are not conflict-free
Admissible extensions

Let $B \subseteq A$. $B$ is an admissible extension iff

1. $B$ is conflict-free
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- $\{a, b\}$ is not an admissible extension
Stable extensions and graph kernels

Let $\mathcal{B} \subseteq \mathcal{A}$. $\mathcal{B}$ is a stable extension iff

1. $\mathcal{B}$ is conflict-free
2. $\mathcal{B}$ attacks any argument in $\mathcal{A} \setminus \mathcal{B}$
Stable extensions and graph kernels

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\[
\begin{array}{c}
a \iff b \\
\end{array}
\]
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- A **kernel** of a (di)graph $G = (V, E)$ is a set $K \subseteq V$ such that
  1. $\forall v_i, v_j \in K$ it holds that $(v_i, v_j) \notin E$ and $(v_j, v_i) \notin E$
  2. $\forall v_i \notin K, \exists v_j \in K$ such that $(v_j, v_i) \in E$
Stable extensions and graph kernels

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Introduced by Von Neumann and Morgenstern in 1944
Stable extensions of $T$ correspond exactly to the kernels of the associated graph $\mathcal{G}_T$ (Dimopoulos+Torres 1996)
Stable extensions and graph kernels

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- A graph may have one or many kernels...
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A graph may have one or many kernels...

...or no kernels at all
Stable extensions and graph kernels

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- A graph may have one or many kernels...

- ...or no kernels at all

- Reasoning with stable/admissible extensions is hard
  - Deciding the existence of stable extensions is NP-hard
  - Deciding the existence of a non-empty admissible extension is NP-hard
Preference-based Argumentation

- An extension of classical argumentation
  
  **Basic Idea:** We often have preferences over arguments
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- Example
  
  - Small cars have low running cost
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- Small cars have low running cost
- Big cars are safe
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- Example
  
  - Small cars have low running cost
  
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  - Safety is more **important** than running cost
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  - Small cars have low running cost
  - Big cars are safe
  - Safety is more *important* than running cost

- Preferences present in previous works on argumentation
  But no systematic study
An extension of classical argumentation

**Basic Idea:** We often have preferences over arguments

**Example**
- Small cars have low running cost
- Big cars are safe
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Preferences present in previous works on argumentation
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**This work:** Study the properties of a specific Preference-based Argumentation Framework
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- A **conflict** relation, $\mathcal{C}$, capturing incompatibility between arguments
- A **preference** relation, $\succ$, capturing the relative strength of arguments
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a \succeq b \text{ means } a \succeq b \text{ and } b \not\succeq a
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Abstract Preference-based Argumentation

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- $\mathcal{C}$ is assumed irreflexive and symmetric
The attacking relation $\mathcal{R}$ is the combination of:
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$a \bowtie b$ means $a \succeq b$ and $b \not\succeq a$

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$\succeq$ is assumed reflexive and transitive, i.e. a pre-order
Abstract Preference-based Argumentation

- The attacking relation $\mathcal{R}$ is the combination of:
  - A *conflict* relation, $\mathcal{C}$, capturing incompatibility between arguments
  - A *preference* relation, $\succeq$, capturing the relative strength of arguments

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- $\mathcal{C}$ is assumed *irreflexive* and *symmetric*
  $\succeq$ is assumed *reflexive* and *transitive*, i.e. a pre-order

- A Preference-based Argumentation Theory (*PBAT*) is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$:
  - $\mathcal{A}$ = a set of arguments
  - $(a, b) \in \mathcal{R}$ iff $(a, b) \in \mathcal{C}$ and $b \not\succ a$
$A = \{ a, b, c \}$
$A = \{a, b, c\}$

$C = \{(a, b), (b, a), (a, c), (c, a)\}$
\[ A = \{a, b, c\} \]
\[ C = \{(a, b), (b, a) (a, c), (c, a)\} \]
\[ a \succ b, \ a \succ c \]
\[ b \succeq c, \ c \succeq b \]
Preference-based Argumentation - Example

- $A = \{a, b, c\}$
- $C = \{(a, b), (b, a), (a, c), (c, a)\}$
- $a \succ b, a \succ c$
  
  $b \succeq c, c \succeq b$
The (di)graph $\mathcal{G}_T$ of a PBAT $T$ has some useful properties.
The graph of PBATs

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- The (di)graph $\mathcal{G}_T$ of a PBAT $T$ has some useful properties
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The (di)graph $G_T$ of a PBAT $T$ has some useful properties:
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Duchet, 1979: kernels *always exist* for certain classes of graphs.
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From those (and other) results we obtain the following properties
- Every PBAT has at least one stable extension
- Every PBAT is coherent
  i.e. stable and maximal admissible extensions coincide
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- From those (and other) results we obtain the following properties
  - Every PBAT has at least one stable extension
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    - i.e. stable and maximal admissible extensions coincide

- All results are based on transitivity
Preferences on sets of arguments

From a preference relation on arguments (\(\succ\)) to a preference relation on sets of arguments:
From a preference relation on arguments ($\succeq$) to a preference relation on sets of arguments:

For $A_1, A_2$ set of arguments, $A_1 \triangleright A_2$ iff
- $A_1 \supset A_2$, or
- $\forall a, b$ with $a \in A_1 \setminus A_2$ and $b \in A_2 \setminus A_1$, it holds that $a \succ b$
Preferences on sets of arguments

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stable extensions = most preferred sets wrt $\triangleright$ permitted by $C$
Preferences on sets on arguments - Example

- $A = \{a, b, c\}$
- $C = \{(a, b), (b, a), (a, c), (c, a)\}$
- $a \succ b$, $a \succ c$
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Preferences on sets on arguments - Example

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Preferences on sets on arguments - Example

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\( a \succ b, a \succ c \)

\( b \succeq c, c \succeq b \)
\{a\} is the unique stable extension

\begin{itemize}
\item \{a\} is the unique stable extension
\end{itemize}
Computing a Stable Extension is Easy

- A stable extension of a PBAT can be computed in \textit{polynomial time}.
Computing a Stable Extension is Easy

- A stable extension of a PBAT can be computed in polynomial time

**General Idea** of the algorithm:
- Start from a top component
- Find an argument that defends itself against all its attackers
- Add the argument to the stable extension and simplify
- Repeat on the remaining theory
Computing a Stable Extension is Easy

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- **General Idea** of the algorithm:
  - Start from a top component
  - Find an argument that defends itself against all its attackers
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  - Repeat on the remaining theory

- **Key property**: There **always exists** a "self-defending" argument
Goal Reasoning is Hard

- Deciding whether there is a stable extension that contains $a$ is NP-hard

  Reduction from 3SAT
Goal Reasoning is Hard

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- Why
  - Complex interaction between arguments
  - Must find the right combination of other arguments
Goal Reasoning is Hard

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Reduction from 3SAT

- Why
  - Complex interaction between arguments
  - Must find the right combination of other arguments

- Deciding whether \( a \) is included in every stable extension is co-NP-hard
Reasoning becomes a bit easier if there is no incomparability.
Theories without incomparability

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  i.e. there are no $a, b \in A$ s.t. $a \not\succeq b$ and $b \not\succeq a$
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- Key Properties
  - Correspondence between the stable extensions of \( T \) and Maximal Independent Sets of \( G_T \)
  - Maximal Independent Sets can be computed with Polynomial Delay
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Key Properties

- Correspondence between the stable extensions of $T$ and Maximal Independent Sets of $G_T$

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- The stable extensions can be computed with Polynomial Delay
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- Key Properties
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  - Maximal Independent Sets can be computed with Polynomial Delay

- The stable extensions can be computed with Polynomial Delay

- Exponential worst case behavior
  A theory with $n$ arguments can have $n^{n/3}$ stable extensions
**Negotiation**: search for a mutually acceptable agreement between two (or several agents) on one or more issues.

- **Offers** ranked by their utility

  - **Reservation value**

- **Alternate Offers Protocol**
**Negotiation**

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- **Characteristics of Negotiation**
  - Deadline?
Negotiation

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- **Alternate Offers Protocol**

**Characteristics of Negotiation**

- Deadline?

- Can I accept an offer that I have previously rejected?

- Issue by issue?
Offers supported by arguments

Argument preference determines offer preference

Best offer is supported by the most preferred argument

Performatives: Propose, Argue, Reject, Agree, Nothing, Withdraw....